Coupling of electromagnetic waves and superlattice vibrations in a piezomagnetic superlattice: Creation of a polariton through the piezomagnetic effect

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We studied the propagation of an electromagnetic (EM) wave in a piezomagnetic superlattice with piezomagnetic coefficient being modulated. Because of the piezomagnetic effect, the coupling between the EM wave and vibration of superlattice can be established, resulting in the creation of a type of magnetic polariton that does not exist in ordinary magnetic material. At some resonance frequencies, the abnormality of dispersion of permeability introduces negative value and piezomagnetic superlattice can make a kind of negative permeability material.

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The concept of negative permeability μ is of particular interest, not only because this is a regime not observed in ordinary materials, but also because such a medium can be combined with a negative permittivity ε to form a "left-handed" material (i.e., $\vec{E}\times\vec{H}$ lies along the direction of $-\vec{k}$ for propagating plane waves).[1–3] In recent work,[4] Pendry *et al.* have introduced the split ring resonator (SRR) medium whose dominant behavior can be interpreted as having an effective magnetic permeability. By making the constituent units resonant, the magnitude of μ is enhanced considerably, leading to a large positive effective μ near the low-frequency side of the resonance and, most strikingly, negative μ near the high-frequency side of the resonance. More recently, a magnetic response at terahertz frequencies has been achieved in a plannar structure composed of SRR elements.[5]

On the other hand, in artificial composites such as superlattices, the periodic modulation of related physical parameters may result in some coupling effects. For example, associated with the variation of dielectric constants is the photonic crystal[6–8] and the modulation of nonlinear optical coefficients results in a quasi-phase-matched frequency conversion.[9,10] Recently, a coupling between the superlattice vibrations and the electromagnetic (EM) wave was established in piezoelectric superlattices,[11–13] in which the piezoelectric coefficient is modulated. This coupling produced a new type of polariton, in which the resonance frequency was determined by the period of superlattice and the negative effective permittivity $\varepsilon_{\text{eff}}(\omega)$ can be got near the high-frequency side of the resonance.

In the present work, we will construct another kind of periodic structure, the piezomagnetic superlattice, with the piezomagnetic coefficient being modulated. In this system, the superlattice vibrations will induce spin waves due to the piezomagnetic effect. The lateral spin waves in turn will emit EM waves that interfere with the original EM wave. In such a case, the lattice vibration will couple strongly with the EM wave and result in polariton excitation. Near the piezomagnetic polariton resonance, a negative $\mu_{\text{eff}}(\omega)$ will be shown possible. Therefore, a kind of negative permeability material can be constituted in piezomagnetic superlattice.

In order to elucidate the above idea, let us consider a one-dimensional (1D) periodic structure composed of alternating layers of $NiOFe_2O_3$ and $NiO_{0.8}ZnO_{0.2}Fe_2O_3$ along the z axis. Through changing the doping contents of NiO and ZnO, a period superlattice can be attained which is shown in Fig. 1. As illustrated in Table I,[14] a different doping ratio introduces a different piezomagnetic coefficient. Therefore, the piezomagnetic coefficient is modulated periodically in the superlattice along the z axis. The piezomagnetic tensor matrix has the form

$$(q_{iJ}) = \begin{bmatrix} 0 & 0 & 0 & 0 & q_{15}(z) & 0 \\ 0 & 0 & 0 & q_{15}(z) & 0 & 0 \\ q_{31}(z) & q_{31}(z) & q_{33}(z) & 0 & 0 & 0 \end{bmatrix}. \quad (1)$$

Here, Voigt's notation is used in the representation of three-order tensor $q_{ijk}\rightarrow q_{iJ}$ $(j,k\rightarrow J)$, which is specified in Table II. In the layers $NiOFe_2O_3$ $(0\leqslant z<d)$, $q_{33}(z)=-212.4$ N/Am and $q_{31}(z)=-96.7$ N/Am, while in the layers $NiO_{0.8}ZnO_{0.2}Fe_2O_3$ $(d\leqslant z<\Lambda)$, $q_{33}(z)=-285.6$ N/Am and $q_{31}(z)=-125.7$ N/Am. The modification is along the z

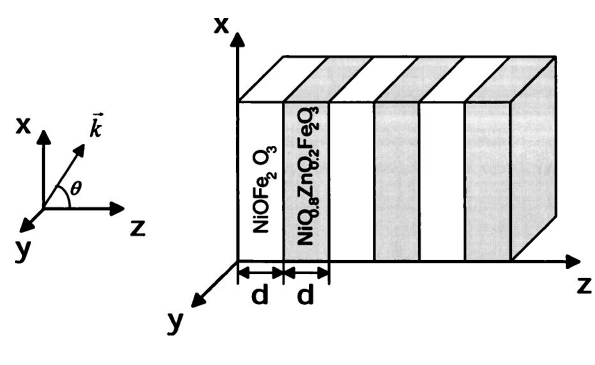

FIG. 1. Schematic illustration of the piezomagnetic superlattice.

TABLE I. Piezomagnetic properties of the $NiO_xZnO_{1-x}Fe_2O_3$ superlattice.

NiO, ZnO, Fe_2O_3	15:35:50	18:32:50	25:25:50	32:18:50	40:10:50	50:0:50
q_{33} (N/Am)	−113.1	−78.5	−128.2	−144.5	−212.4	−285.6
q_{31} (N/Am)	−44.1	−31.8	−61.9	−79.5	−96.7	−125.7

axis; the reciprocal vectors for periodic structure should be $\vec{G}=(0,0,G_m)$ $(G_m=m\pi/d, m=1,3,5,\ldots)$. Also we assume that the transverse dimensions are very large compared with an acoustic wavelength so that a one-dimensional model is applicable. When an EM wave is radiated into this piezomagnetic superlattice, an acoustic wave will be excited through the piezomagnetic effect. The coupled interaction between the EM wave and acoustic wave can be described by the constitutive equations

$$T_{ij} = c_{ijkl}\frac{\partial u_k}{\partial x_l} + q_{ijk}(z)H_k, \quad B_i = \mu_{ij}^S H_j - q_{ijk}(z)\frac{\partial u_j}{\partial x_k}, \quad (2)$$

where T_{ij}, $\vec{u}$, $\vec{H}$, $\vec{B}$, c_{ijkl}, and μ_{ij}^s are the stress tensor, displacement field, magnetic field, magnetic displacement, and the elastic and static permeability, respectively. With the use of Newton's law $\rho\partial^2 u_j/\partial t^2=(\partial/\partial x_i)T_{ij}$, the equation of motion for a vibrating medium can be obtained

$$\rho\frac{\partial^2 u_j}{\partial t^2} - c_{ijkl}\frac{\partial^2 u_k}{\partial x_i \partial x_l} = \frac{\partial(q_{ijk}(z)H_k)}{\partial x_i}, \quad (3)$$

where ρ is the mass density and c_{ijkl} is the stiffness tensor of superlattice. By using the Fourier transformation

$$u_j = \int \tilde{u}_j e^{i(\omega t - \vec{q}\cdot\vec{r})} d\vec{q}, \quad H_k \int \tilde{H}_k e^{i(\omega t - \vec{k}.\vec{r})} d\vec{k},$$

$$q_{ijk}(z) = \sum_{\vec{G}} \tilde{q}_{ijk}(\vec{G}) e^{-i\vec{G}\cdot\vec{r}}, \quad (4)$$

where $\vec{q}$ is the phonon wave vector, $\vec{k}$ is the electromagnetic wave vector, and $\vec{G}$ is the reciprocal vector of modulated structures, Eq. (2) can be expressed as

$$\int (-\rho\omega^2\delta_{jk} + c_{ijkl}q_i q_l)\tilde{u}_k e^{i(\omega t - \vec{q}\cdot\vec{r})} = \sum_{\tilde{G}} \int (-i)(\vec{k}+\vec{G})_i \tilde{q}_{ijk}\tilde{H}_k e^{i[\omega t - (\vec{k}+\vec{G})\cdot\vec{r}]} d\vec{k}. \quad (5)$$

For photons with long wavelength, $|\vec{k}| \ll \vec{G}$, Eq. (5) becomes

$$\int (-\rho\omega^2\delta_{jk} + c_{ijkl}q_i q_l)\tilde{u}_k e^{i(\omega t - \vec{q}\cdot\vec{r})} = \sum_G (-i) G_i \tilde{q}_{ijk}\tilde{H}_k e^{i[\omega t - (\vec{k}+\vec{G})\cdot\vec{r}]}. \quad (6)$$

In order for the two sides of Eq. (5) to be equal, $\vec{q}$ must satisfy $\vec{q}=\vec{k}+\vec{G}\approx\vec{G}$ for one of the reciprocal vectors of the periodic superlattice. Then we obtain, from Eq. (6),

$$(-\rho\omega^2\delta_{jk} + c_{ijkl}G_i G_l)\tilde{u}_k(\vec{q}=\vec{k}+\vec{G}) = (-i)G_i\tilde{q}_{ijk}\tilde{H}_k \quad (7)$$

and we have

$$\tilde{u}_k = i(\rho\omega^2\delta_{jk} - c_{ijkl}G_i G_l)^{-1} G_i \tilde{q}_{ijk}\tilde{H}_k. \quad (8)$$

Then,

$$\frac{\partial u_k}{\partial x_j} = e^{-i\vec{G}\cdot\vec{r}} G_j (\rho\omega^2\delta_{jk} - c_{ijkl}G_i G_l)^{-1} G_i \tilde{q}_{ijk} H_k(\vec{r},t). \quad (9)$$

Substituting Eq. (9) into (2) and using the space average value, we get

$$B_i = \mu_{ik}(\omega)H_k, \quad (10)$$

where

$$\mu_{ik}(\omega) = \mu_{ik}^S + \tilde{q}_{ikj}G_j(-\rho\omega^2\delta_{jk} + c_{ijkl}G_i G_I)^{-1} G_i \tilde{q}_{ijk}. \quad (11)$$

If we choose the first reciprocal vector $\vec{G}=(0,0,G_1=\pi/d)$, the permeability coefficients matrix can be attained as

$$\vec{\mu}(\omega) = \begin{bmatrix} \mu_\perp(\omega) & 0 & 0 \\ 0 & \mu_\perp(\omega) & 0 \\ 0 & 0 & \mu_\parallel(\omega) \end{bmatrix}. \quad (12)$$

Here,

$$\mu_\perp(\omega) = \mu_\parallel^s \frac{\omega_{\perp,0}^2 - \omega^2}{\omega_\perp^2 - \omega^2}, \quad \mu_\parallel(\omega) = \mu_{33}^s \frac{\omega_{\parallel,0}^2 - \omega^2}{\omega_\parallel^2 - \omega^2}, \quad (13)$$

where $\omega_\perp = G_1\sqrt{c_{44}/\rho}$, $\omega_\parallel = G_1\sqrt{c_{33}/\rho}$, $\omega_{\perp,0} = \sqrt{\omega_\perp^2 + \tilde{q}_{31}^2 G_1^2/(\rho\mu_{11}^s)}$, and $\omega_{\parallel,0} = \sqrt{\omega_\parallel^2 + \tilde{q}_{33}^2 G_1^2/(\rho\mu_{33}^s)}$. $\omega_\perp$ and $\omega_\parallel$ are the resonance frequency of the longitudinal vibration and transverse vibration due to piezomagnetic effect. Equation (13) exhibits the resonance structure produced in the piezomagnetic superlattice which profoundly affects the propagation of electromagnetic waves with frequency near $\omega_\perp$ and $\omega_\parallel$.

The effective dielectric tensor of piezomagnetic superlattice can be derived from the average-field method. Straightforward algebra gives the effective dielectric tensor in the well-known form[15]

TABLE II. Voigt's notation used in representation of $q_{ijk} \to q_{iJ}$.

(j,k)	(1, 1)	(2, 2)	(3, 3)	(2, 3) (3, 2)	(1, 3) (3, 1)	(1, 2) (2, 1)
J	1	2	3	4	5	6

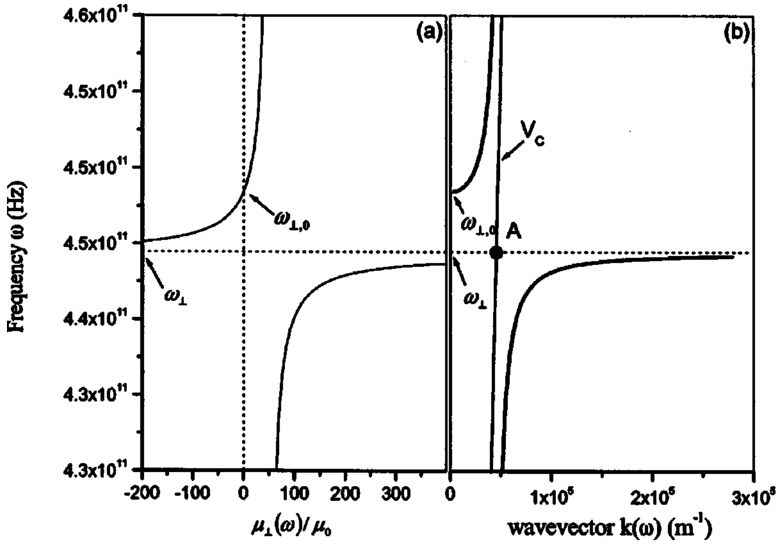

FIG. 2. Calculated magnetic abnormality (a) and polariton disersion (b) of the coupled mode of the photon and pure transverse phonon.

$$\overleftrightarrow{\varepsilon}=\begin{bmatrix}\varepsilon_\perp & 0 & 0\\ 0 & \varepsilon_\perp & 0\\ 0 & 0 & \varepsilon_\parallel\end{bmatrix}. \tag{14}$$

In the discussion that follows, we shall assume for simplicity that the dielectric constant of the superlattice is independent of frequency near the magnetic resonance frequencies $\omega_\perp$ and $\omega_\parallel$.

The dispersion relation for polariton propagating in piezomagnetic superlattices follows from Maxwell's equations. After eliminating $\vec{E}$ in the curl equations, one obtains the following wave equation for $\vec{H}$:

$$\vec{k}\times[\overleftrightarrow{\varepsilon}^{-1}\cdot(\vec{k}\times\vec{H})]+\frac{\omega^2}{c^2}\overleftrightarrow{\mu}(\omega)\cdot\vec{H}=0, \tag{15}$$

where c is the electromagnetic wave velocity in vacuum, ω the angular frequency, and $\overleftrightarrow{\varepsilon}^{-1}$ the inverse of Eq. (14). Since the susceptibility tensor is isotropic in the xy plane, there is no loss of generality if we choose the wave vector $\vec{k}$ in the xz plane, as shown in Fig. 1. The angle between the wave vector and the z axis will be denoted by θ, so that

$$k_x=k\sin\theta,\quad k_z=k\cos\theta. \tag{16}$$

Then, if we proceed by the use of Eq. (15), we have

$$\left(\frac{\omega^2}{c^2k^2}\mu_\perp\varepsilon_\perp-\cos^2\theta\right)H_x+\sin\theta\cos\theta H_z=0, \tag{17a}$$

$$\left(\frac{\omega^2}{c^2k^2}\mu_\perp\varepsilon_\perp-\sin^2\theta\frac{\varepsilon_\perp}{\varepsilon_\parallel}-\cos^2\theta\right)H_y=0, \tag{17b}$$

$$\sin\theta\cos\theta H_x+\left(\frac{\omega^2}{c^2k^2}\mu_\parallel\varepsilon_\perp-\sin^2\theta\right)H_z=0. \tag{17c}$$

Equations (17) are a set of three homogeneous equations satisfied by $\vec{H}$ in the superlattice. The polariton dispersion relation due to the coupling between the spins and electromagnetic wave can be obtained by setting the determinant in Eqs. (17) equal to zero.

From Eqs. (17), one sees that there are two propagating modes: one with the magnetic field vector normal to the xz plane and one with the magnetic field in the xz plane. Considering the first mode with the magnetic field normal to the xz plane, the magnetic field is in the y direction, parallel to the xy plane. We refer to this mode as the TM mode. This dispersion relation is obtained from Eqs. (17b) and (13):

$$\frac{c^2k^2}{\omega^2}=\mu_{11}^s\varepsilon(\theta)\frac{\omega_{\perp,0}^2-\omega^2}{\omega_\perp^2-\omega^2}, \tag{18}$$

where $\varepsilon(\theta)=\sqrt{\sin^2\theta/\varepsilon_\parallel+\cos^2\theta/\varepsilon_\perp}$. The polariton with the magnetic field in the xz plane has a more complex dispersion relation for a general value of θ. We refer to this second mode as the TE polariton. When $\theta=0$ (propagation along z axis), the dispersion relation is

$$\frac{c^2k^2}{\omega^2}=\mu_{11}^s\varepsilon_\perp\frac{\omega_{\perp,0}^2-\omega^2}{\omega_\perp^2-\omega^2}. \tag{19}$$

It has same resonance frequency with that obtained for the TM polariton, which described the coupling between the EM mode, $V_c=c/\sqrt{\mu_{11}^s\varepsilon_\perp}$, and the transverse phonons. When $\theta=\pi/2$ (propagation along the x axis), the dispersion relation becomes

$$\frac{c^2k^2}{\omega^2}=\mu_{33}^s\varepsilon_\perp\frac{\omega_{\parallel,0}^2-\omega^2}{\omega_\parallel^2-\omega^2}, \tag{20}$$

which shows coupling happens between the EM mode $V_c=c/\sqrt{\mu_{33}^s\varepsilon_\perp}$ and the longitudinal phonons. For any other direction ($\theta\neq0$ and $\pi/2$), the resonance frequency is neither $\omega_\perp$ nor $\omega_\parallel$ and no pure longitudinal and transverse coupling occurs. EM wave is coupled to the admixture mode of longitudinal phonon and transverse phonon.

Figures 2(a) and 2(b) shows the coupled modes of photon and transverse phonons in the superlattice described by Eqs. (13) and (19). The solid line labeled $V_c=c/\sqrt{\mu_{11}^s\varepsilon_\perp}$ corresponds to EM waves, but uncoupled to the lattice vibrations,

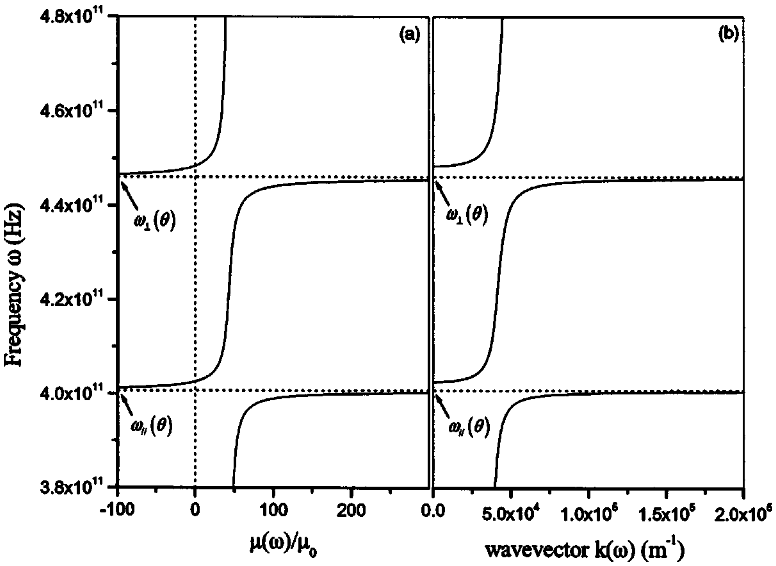

FIG. 3. Calculated magnetic abnormality (a) and polariton disersion (b) of the coupled mode of the photon and admixture of the longitudinal phonon and transverse phonon.

and the dotted line represents the lattice vibration in the absence of coupling to the EM field. The region of the crossover [marked by A in Fig. 3(b)] of the solid line and the dotted line is the resonance region. By resonance, we mean that the frequency of the EM wave equals the acoustic resonance frequency of the superlattice determined by the periodicity. At resonance the photon-phonon coupling entirely changes the character of the propagation. The heavy lines are the dispersion relations in the presence of coupling between the lateral spins induced by a lattice vibration and the EM wave. In the resonance region the propagation mode is neither a pure photon mode nor a pure acoustic mode in a narrow range of k values. The quantum of the coupled photon-phonon wave field is called a polariton. It is a type of polariton. One effect of the coupling is to create a frequency gap between $\omega_\perp$ and $\omega_{\perp,0}$. For frequencies $\omega_\perp < \omega < \omega_{\perp,0}$, $\mu(\omega)$ is negative as can be seen in Fig. 3(a). This negative gap originates from the coupling of the photon and the lattice vibration through the piezomagnetic effect. The resonance frequency $\omega_\perp = (\pi/d)\sqrt{c_{44}/\rho}$ is mainly determined by the periodic of the superlattice, $\Lambda = 2d$. As $\omega_\perp < \omega_{\perp,0}$, the band gap is $\omega_{\perp,0} - \omega_\perp$ wide, which is determined by $\Delta^2 = \tilde{q}_{31}^2 G_1^2/(\rho\mu_{11}^s)$. The larger the Δ value is, the stronger the coupling and the wider the gap. The negative band can be widened by use of some materials with larger piezomagnetic coupling coefficients. If the period is $\Lambda = 0.1$ μm, the first-order reciprocal vector of this periodic structure can be attained as $G_1 = \pi/d = 6.283 \times 10^7$ m^{-1}. The average mass density of superlattice is $\rho = 5.0 \times 10^3$ kg/m^3 and the piezomagnetic parameters used is given in Table I. The transverse resonance frequency can be attained as $\omega_\perp = 4.44 \times 10^{11}$ Hz and the negative range is about 0.5×10^{10} Hz. By varying the period of the superlattice, a wide range of negative permeability can be achieved.

If the EM wave is radiated into the piezomagnetic superlattice with an oblique angle ($\theta \neq 0$ and $\pi/2$), the EM wave will not only couple with the longitudinal phonon but also couple with the transverse phonon. The dispersion relation of the admixture polariton can be calculated from Eqs. (17a) and (17c), which is shown in Figs. 3(a) and 3(b). It can be seen that there are two resonance regions in the curves. But the resonances are not equal to $\omega_\perp$ and $\omega_\parallel$, which are both functions of θ. In this general case, negative permeability can also be attained in the two resonance regions. The above results show that the piezomagnetic superlattice can be a kind of artificial material to produce negative permeability.

In this paper, we only consider one-dimensional piezomagnetic multilayer structure. The mathematical model introduced here can be extended to two- and three-dimensional structures, which can also be proved to produce negative permeability. On the other hand, the piezomagnetic superlattice can be combined with the piezoelectric superlattice to make left-handed material. Further study will be carried out to realize this object.

In summary, a kind of periodic structure, the piezomagnetic superlattice, is constructed in this paper. The propagation property of an EM wave in a piezomagnetic superlattice was studied theoretically. A type of magnetic polariton was proposed which originates from the coupling of the EM wave with the lattice vibration through the piezomagnetic effect. At some resonance frequency regions, negative permeability can be attained and the piezomagnetic superlattice can make a kind of negative permeability material.

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[1] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[2] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[3] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[4] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
[5] T. J. Yen, W. J. Padila, N. Fang, D. C. Vier, D. R. Smith, J. B. Pendry, D. N. Basov, and X. Zhang, Science **303**, 1494 (2004).
[6] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[7] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[8] J. J. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals* (Princeton University, Princeton, NJ, 1995).
[9] S. N. Zhu, Y. Y. Zhu, and N. B. Ming, Science **278**, 843 (2004).
[10] H. Liu, Y. Y. Zhu, S. N. Zhu, C. Zhang, and N. B. Ming, Appl. Phys. Lett. **79**, 728 (2001).
[11] Y. Q. Lu, Y. Y. Zhu, Y. F. Chen, S. N. Zhu, N. B. Ming, and Y. J. Feng, Science **284**, 1822 (1999).
[12] Y. Y. Zhu, X. J. Zhang, Y. Q. Lu, Y. F. Chen, S. N. Zhu, and N. B. Ming, Phys. Rev. Lett. **90**, 053903 (2003).
[13] X. J. Zhang, R. Q. Zhu, J. Zhao, Y. F. Chen, and Y. Y. Zhu, Phys. Rev. B **69**, 085118 (2004).
[14] Z. G. Zhou, *Ferrite Magnetic Materials* (Science Publishing House, Beijing, 1981).
[15] V. M. Agranovich and V. E. Kravstov, Solid State Commun. **55**, 85 (1985).