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Negative refraction with magnetic resonance in a metallic double-ring metamaterial

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Metallic metamaterials, by periodically arranged double rings, without splits in both of them, are investigated to show their negative refraction at the magnetically resonant transmission band. It is verified that the negative refraction occurs in the transmission regime where the electric response is attributed to the plasma oscillation of electrons, in analogy to the metallic cut-wire metamaterials, and the magnetic response is originated from antiparallel currents induced resonantly in the neighboring edges of the concentric double rings. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2837186]

Left-handed materials (LHMs) and their negative refraction phenomena are of great interest for the simultaneously negative electric and magnetic responses (i.e., effective permittivity $\varepsilon_{\mathrm{eff}}<0$ and effective permeability $\mu_{\mathrm{eff}}<0$).[1–3] Various metallic artificial structures, such as the double split-ring resonators (DSRRs),[4,5] split-ring chains,[6] and cut-wire pairs,[7,8] have been proposed in pursuing the negative refraction and/or magnetic response above gigahertz frequencies. Generally speaking, no matter what geometry configurations they are, a common viewpoint of *LC* equivalent circuit is reliable in interpreting the underlying electromagnetic mechanism,[8,9] where the inductance *L* comes from induced currents and a gap or split stands for the capacitance *C*. As far as the magnetic response of DSRRs is concerned, it is the split introduced in the ring that makes a nonmagnetic metallic element act as a subwavelength magnetic "atom." As a matter of fact, single split-ring resonators (SSRRs) can keep magnetic effect approximately in the same way of DSRRs.[9] However, for a single ring without split, no magnetic effect is expected in the long wavelength regime. As for the magnetic metamaterial comprising cut-wire pairs, the antisymmetric resonant mode is responsible for its magnetic effect, corresponding to antiparallel currents in the interacting pairs.[10,11]

However, it is well known that for the single-split DSRRs or SSRRs as usually used, the in-plane isotropy is destroyed by the split (and cut-wire pairs as well) and, hence, different propagation directions along the ring edges (with or without splits) would lead to different electromagnetic responses,[12] in some cases resulting in the undesired bianisotropic effect, which causes rather complex circumstance for the magnetic response. On the other hand, though the negative-μ_{eff} DSRRs is believed to have negative electric response in certain frequency range,[13] it is difficult to realize simultaneously negative $\varepsilon_{\mathrm{eff}}$ and μ_{eff} in the metamaterial composed of DSRR units.[11,14]

In this letter, we investigated a concentric double-ring metamaterial which only differs from the DSRRs in that there are no splits introduced in both of the double rings. It is found that the proposed metamaterial can generate negative refraction in the magnetically resonant transmission regime. The simplification of the double-ring metamaterial, with respect to the DSRRs, is of benefit not only for its convenience in fabrication, but also for its isotropic electromagnetic response in the ring plane.

Figure 1 shows schematically the unit cell of the double-ring metamaterial, where the rings are copper in square with a thickness of 0.02 mm and the geometric parameters *a* and *b* are 3.0 and 0.2 mm, respectively. The gap between the inner and outer rings, represented by *g*, is 0.1 mm. Note that the square shape of the rings does not introduce qualitative difference with respect to a circular shape. In our numerical simulations based on the full-wave finite element method, the simulation configuration has a dimension of $1\times1\times8$ units (i.e., one unit in the transverse *xy* plane and eight units in the electromagnetic propagation direction along the *z* axis). The polarized incident waves with electric field in the

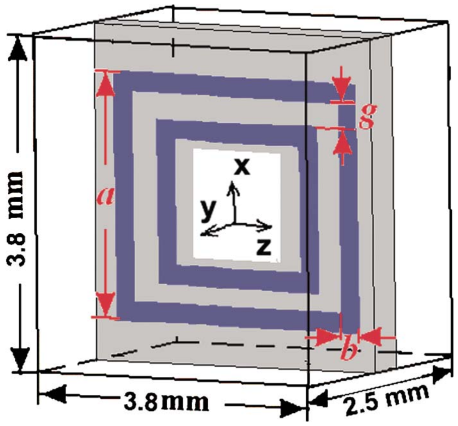

FIG. 1. (Color online) Unit cell of the metallic double-ring metamaterial.

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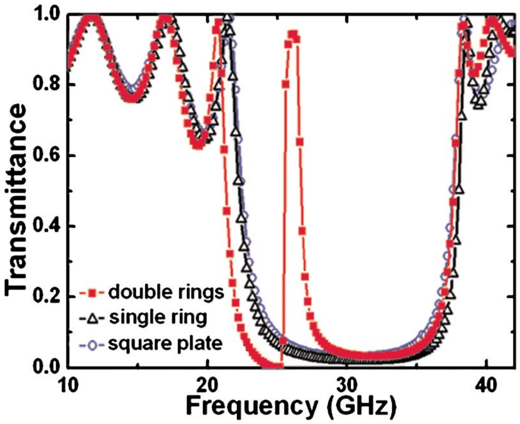

FIG. 2. (Color online) Transmission spectra of the double-ring, single-ring, and square-plate metamaterials.

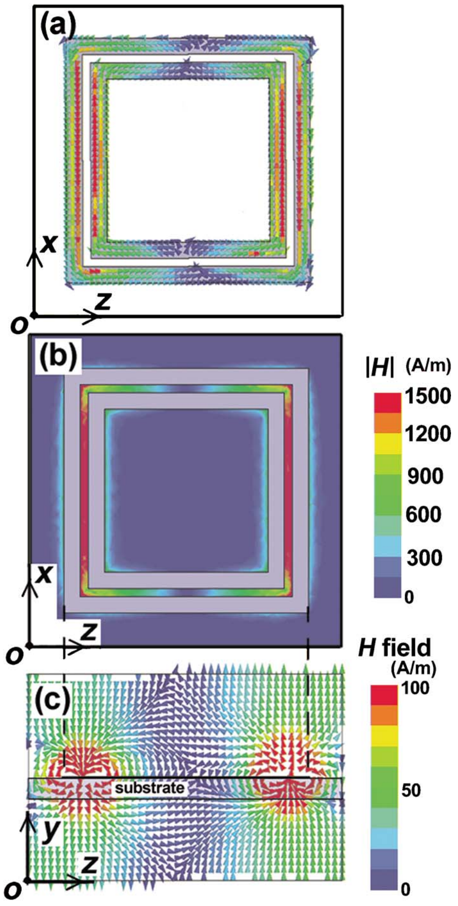

FIG. 3. (Color online) Magnetic resonance at 26 GHz. (a) Distribution of induced currents on the surface of the double rings. (b) The magnitude distribution of the magnetic field in *xz* plane. (c) Magnetic field map in the *yz* plane with *x* position at the midpoints of the ring edges.

x direction and magnetic field in the *y* direction are satisfied by applying the perfect electric and magnetic boundary conditions, respectively.[15,16] To simulate a realistic system, the rings are designed onto a 0.25-mm-thick Teflon substrate with relative dielectric constant of 2.1.

The transmittance of the double-ring metamaterial is shown in Fig. 2 (red square line). It is interesting to find that there is a resonant transmission peak at about 26 GHz (this peak will shift to around 30 GHz if no Teflon substrate is considered). First of all, it is noticed that the resonant peak resembles the left-handed peak of a combined metamaterial by the metallic wires and DSRRs.[17,18] Concretely speaking, the resonant peak is located at the right side of the resonant frequency. This should indicate a potential "left-handed" peak since the effective constitutive parameters are generally only negative above and next to the resonant frequencies (i.e., right side). To make further investigations on the transmission spectrum of the proposed double-ring metamaterial, we can simulate two relative structures. One is to modify the double-ring geometry to be a single-ring configuration (here, set $g=0$), the other is to fill up the single rings into square plates, each plate with the same cross section $a\times a$ as a double-ring structure. From the simulation results shown in Fig. 2, it is obvious that the transmission of the single-ring metamaterial (black triangle line) approximately coincides with the square-plate case (blue circle line). In fact, the latter essentially resembles a cut-wire metamaterial, with its ε_{eff} written as $1-(\omega_{\text{ep}}^2-\omega_{\text{eo}}^2)/(\omega^2-\omega_{\text{eo}}^2)$.[13,19] Therefore, the stop band in the frequency range approximately from 21 to 38 GHz, corresponding to ω_{eo} and ω_{ep}, respectively, should be attributed to negative ε_{eff} (if applicable). Therefore, the transmission band around 26 GHz of the double-ring structure is occurred within a stop band due to negative electric response. In the next part, we will verify that this transmission band is originated from a magnetic resonance and can exhibit negative refraction.

As is well known, to design a metamaterial with sufficient magnetic response beyond gigahertz frequencies usually relies on the circulating currents, which generates a magnetic moment opposed to the incoming magnetic field. Therefore, to confirm the magnetic characteristic of the resonant transmission around 26 GHz, we can explore the distribution of its induced currents. It is found, interestingly, that the two neighboring vertical edges of the inner and outer rings have antiparallel directions of their induced currents [see Fig. 3(a), 26 GHz], and it is the gap between the inner and outer rings that makes the resonance possible. The antiparallel current directions are in literatures called antisymmetric resonant mode,[10,11] which contributes to negative magnetic response. Figure 3(b) shows the magnitude distribution of the magnetic field, strongly concentrated between the neighboring edges with antiparallel induced currents. From the $\bar{\boldsymbol{H}}$ field map shown in Fig. 3(c), the magnetic resonance of the double-ring structure is not a simple duplication of the cut-wire pairs because the left and right gaps of the double rings have different magnetic directions due to the opposite circulations of corresponding current distributions. Overall, the coupled magnetic alignments between the two sides form a counterclockwise rotation so that they can be strongly concentrated in the left and right gaps and mainly canceled each other only in the middle part of the double rings.

As mentioned before, we have verified the resonant transmission is electrically effected in analogy to negative ε_{eff} of a cut-wire metamaterial and is magnetically effected in antisymmetric resonant mode which usually results in negative μ_{eff}. It should be expectable to characterize this resonant transmission with simultaneous negative ε_{eff} and μ_{eff} by the retrieval procedure.[20,21] However, the unit cell of the proposed structure is comparable in sizes with the incident wavelength of the resonant frequency and, thus, the applicability of the effective medium theory is questionable. Nevertheless, the negative refraction of the resonant trans-

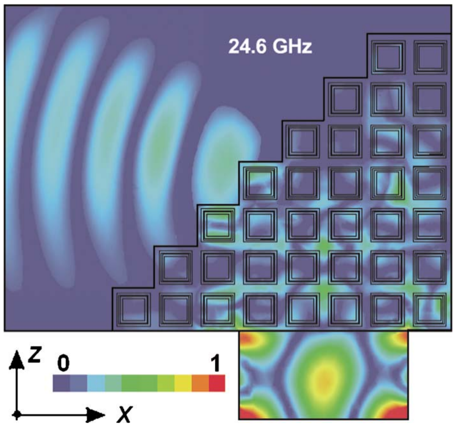

FIG. 4. (Color online) Magnitude distribution of the magnetic field for the wedge-shaped negative refraction of the double-ring metamaterial at frequency of 24.6 GHz.

mission can be verified by a wedge-shaped configuration of the double-ring metamaterial (see Ref. 16 for more details about the simulation conditions). As is found in Fig. 4, unambiguous negative refractions are observed in the resonant transmission band. Three aspects should be noted for Fig. 4. (1) It is plotted in a plane away from the metallic elements in order to avoid serious resonant concentration of the field around the metallic elements. (2) The frequency shown in Fig. 4 is 24.6 GHz, a slight shift out of the left-handed band (see Fig. 2, red square line). This discrepancy is caused because periodically arranged unit cells are straightforwardly used in the simulation of wedge-shaped negative refraction, while in calculating the transmittance; instead, suitable boundary conditions are sufficient to mimic the periodic configuration. (3) It would be mentioned that positive refractions in the passbands below 21 GHz as well as above 38 GHz are observed in our simulations (not shown here).

In summary, the metallic metamaterial composed of periodically arranged double rings is verified to have a resonant transmission with negative refraction as well as magnetic resonance. In the negatively refracted regime, the electric response is confirmed to be in analogy to the cut-wire metamaterials and the magnetic resonance is caused by an antisymmetric resonant mode of induced current. It should be mentioned that, in similar to many other metallic metamaterials, an equivalent *LC* circuit model can be used to analyze the resonant frequency of the double-ring metamaterial, which is not involved in this letter for length limit.

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