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Lamellar model of the left-handed metamaterials composed of metallic split-ring resonators and wires

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ABSTRACT

Left-handed materials composed of split ring resonators (SRRs) and wires are investigated in the viewpoint of lamellar composite with epsilon-negative (ENG) and mu-negative (MNG) materials staked alternatively. Several configurations of the SRR-wire metamaterial are numerically simulated to confirm its left-handed response as an analogy to the ENG–MNG lamellar model.

1. Introduction

Metallic metamaterials are the main solutions to realize the left-handed materials (LHMs), which have evoked considerable attention in recent years for their significant electromagnetic characteristics [1–4]. Up to date, in addition to the metallic metamaterial comprising split ring resonators (SRRs) and wires (hereafter, SRR-wire metamaterial) [3], various metallic metamaterials in different shapes have been claimed to be LHMs, such as the Ω-shaped [5], S-shaped [6], H-shaped [7], and double-wire metamaterials [8,9]. Amongst all of these metallic LHMs, SRR-wire metamaterial is of importance not only for its first realization of LHMs, but also for the facts that almost all of the other metallic metamaterials claimed to be LHMs are designed according to the SRR-wire prototype. However, despite that the SRR-wire metamaterial has been confirmed experimentally as well as numerically to be LHM [3,10], the physical mechanism for the origin of its left-handed (LH) response, or how to interpret its effective constitutive parameters is not satisfactorily clarified yet.

As is known, the original idea for designing the SRR-wire metamaterial was to obtain an LHM with negative $\varepsilon_{\mathrm{eff}}$ of the wire array and negative μ_{eff} of the SRR array, and it was supposed that electromagnetic interactions between the SRR and wire arrays were negligible [11,12]. However, this criterion was confronted by some researchers [13], in which the main viewpoint was that an LHM could not be obtained by simply placing wire array ($\varepsilon_{\mathrm{eff}} < 0$) in a homogeneous host with negative μ. More recently, this criterion was confirmed to be incorrect in explaining some experimental results [14]. In the opinion of this work, the SRR-wire LHM can be interpreted as an analogy to the lamellar composite stacked by alternative epsilon-negative (ENG) and mu-negative (MNG) layers [15]. It is found that whether or not SRR-wire metamaterials would result in LHM is subjected to the relative arrangements of the SRRs and the wires.

2. ENG–MNG lamellar model

The recent work has demonstrated that an ENG–MNG lamellar composite can effectively act as LHM under the condition of effective medium approximation [15]. Shortly speaking, consider a lamellar composite as shown in Fig. 1. Two infinite slices, one is an ENG layer ($\varepsilon_1 < 0$, $\mu_1 > 0$, with thickness d_1) and the other an MNG layer ($\varepsilon_2 > 0$, $\mu_2 < 0$, with thickness d_2), are stacked

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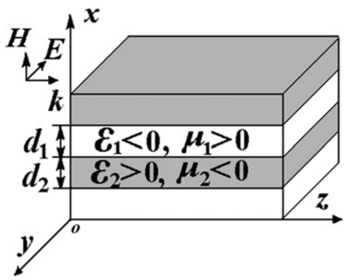

Fig. 1. Schematic diagram of a lamellar composite with ENG and MNG materials stacked alternatively. Only two periods of the lamellar structure are shown.

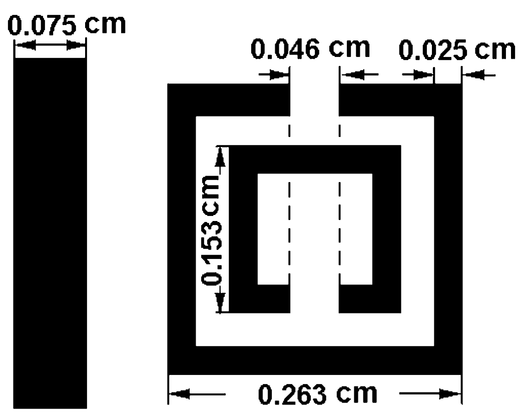

Fig. 2. Scales of the wire and SRR elements. The metal is 0.0017 cm in thickness and the wire is effectively infinite in length for all simulations.

alternatively along the x-axis. Based on the effective medium approximation (i.e., both d_1 and d_2 are extremely smaller than the electromagnetic wavelength), the components of ε and μ can be given by [15,16]

$$\bar{\varepsilon}_x = \frac{\varepsilon_1 \varepsilon_2 (d_1 + d_2)}{\varepsilon_1 d_2 + \varepsilon_2 d_1}, \tag{1}$$

$$\bar{\varepsilon}_y = \bar{\varepsilon}_z = \frac{\varepsilon_1 d_1 + \varepsilon_2 d_2}{d_1 + d_2}, \tag{2}$$

$$\bar{\mu}_x = \frac{\mu_1 \mu_2 (d_1 + d_2)}{\mu_1 d_2 + \mu_2 d_1}, \tag{3}$$

$$\bar{\mu}_y = \bar{\mu}_z = \frac{\mu_1 d_1 + \mu_2 d_2}{d_1 + d_2}. \tag{4}$$

When electromagnetic waves are polarized with electric field in the y-direction and magnetic field in the x-direction, only ε_y and μ_x take effect [17]. From Eqs. (2) and (3), simultaneously negative ε_y and μ_x can be obtained when $|\varepsilon_1| d_1 > \varepsilon_2 d_2$ and $\mu_1 d_2 > |\mu_2| d_1$, and thus such an ENG–MNG lamellar composite can act effectively in the way of LHMs [15,17].

It is the objective of this Letter to interpret the SRR-wire metamaterial as an analogy to the ENG–MNG lamellar model. To make such an interpretation intuitively, we can compare the ENG–MNG model with one-dimensional SRR-wire metamaterials as illustrated in a number of literatures [10,18], where the wire layers can be regarded as ENG material and the SRR layers as MNG material. In the next section, different configurations will be investigated numerically to confirm this analogy, and it is for convenience that simulations in all configurations are based on the same scales of the wire and SRR elements depicted in Fig. 2. In our simulations based on the full-wave finite element method, the SRR and wire elements are copper, which can be approximately regarded as perfect conductor in the microwave spectrum.

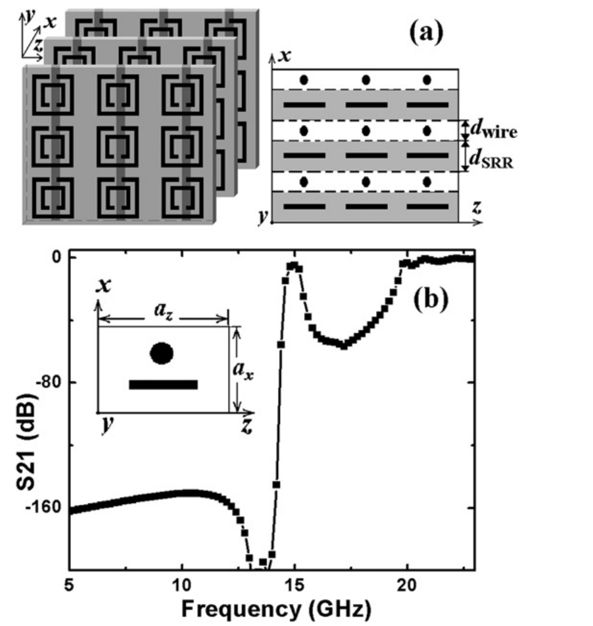

Fig. 3. (a) The off-plane configuration. The right panel is schematically shown to reveal the lamellar analogy, with wires and SRRs denoted by black dots and bars, respectively. (b) Transmission coefficient calculated with the unit cell in dimensions $a_x = 0.33$ cm, $a_y = 0.33$ cm, and $a_z = 0.443$ cm.

3. Different SRR-wire configurations

3.1. Off-plane case

First of all, the generally used configuration [Fig. 3(a)], so-called off-plane case, is simulated to verify the existence of LH peak. As is well known [3,10], such SRR-wire metamaterials exhibit simultaneously negative ε_{eff} and μ_{eff} when electromagnetic waves are polarized with electric field in the y-direction and magnetic field in the x-direction. Under this incidence condition, it is found in Fig. 3(b) that the LH transmission peak is located around 15 GHz (exactly within the resonant band of the SRRs). From the viewpoint of ENG–MNG lamellar model, two mathematic expressions for ε_{eff} and μ_{eff} can be introduced to the off-plane configuration of SRR-wire metamaterials:

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_{\text{SRR}} d_{\text{SRR}} + \varepsilon_{\text{wire}} d_{\text{wire}}}{d_{\text{SRR}} + d_{\text{wire}}} = \frac{\varepsilon_{\text{SRR}} d_{\text{SRR}} / d_{\text{wire}} + \varepsilon_{\text{wire}}}{d_{\text{SRR}} / d_{\text{wire}} + 1}, \tag{5}$$

$$\mu_{\text{eff}} = \frac{\mu_{\text{SRR}} \mu_{\text{wire}} (d_{\text{SRR}} + d_{\text{wire}})}{\mu_{\text{SRR}} d_{\text{wire}} + \mu_{\text{wire}} d_{\text{SRR}}} = \frac{\mu_{\text{SRR}} \mu_{\text{wire}} (d_{\text{SRR}} / d_{\text{wire}} + 1)}{\mu_{\text{SRR}} + \mu_{\text{wire}} d_{\text{SRR}} / d_{\text{wire}}}, \tag{6}$$

where ε_{SRR} and $\varepsilon_{\text{wire}}$ are permittivity components in the direction parallel to the wires (y-axis), while μ_{SRR} and μ_{wire} are permeability components in the direction perpendicular to the SRR-plane (x-axis). The "thicknesses" of the SRR and wire layers are represented by d_{SRR} and d_{wire}, respectively. It is taken for granted that $\varepsilon_{\text{wire}} < 0$ and $\mu_{\text{SRR}} < 0$ in the resonant frequency regime of the SRRs.

From Eqs. (5) and (6), the SRR-wire metamaterial can act as LHMs only if $\varepsilon_{\text{SRR}} d_{\text{SRR}} < |\varepsilon_{\text{wire}}| d_{\text{wire}}$ and also $|\mu_{\text{SRR}}| d_{\text{wire}} < \mu_{\text{wire}} d_{\text{SRR}}$. These two relations mean that, with the approximation of $d_{\text{SRR}} \approx d_{\text{wire}}$, sufficiently negative $\varepsilon_{\text{wire}}$ is required, and in contrast μ_{SRR} is required to be slightly negative. The former requirement is consistent with Ref. [19]. As for the latter, the following in-plane configuration will offer an evidence to support it.

3.2. Wire-shifting case

The wire-shifting configuration is investigated with the shifting distance $s = 0.075$ cm (see the inset of Fig. 4, this case is identical

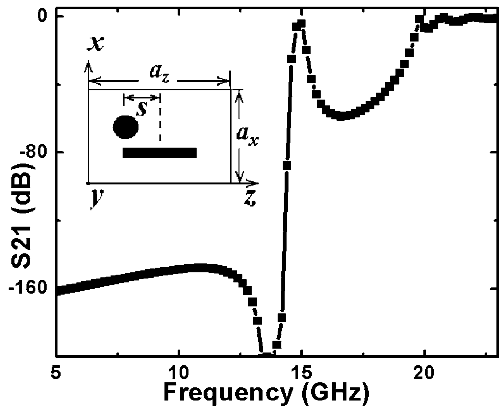

Fig. 4. Transmission coefficient of the wire-shifting configuration, calculated with the unit cell in dimensions $a_x = 0.33$ cm, $a_y = 0.33$ cm, $a_z = 0.443$ cm, and $s = 0.0075$ cm.

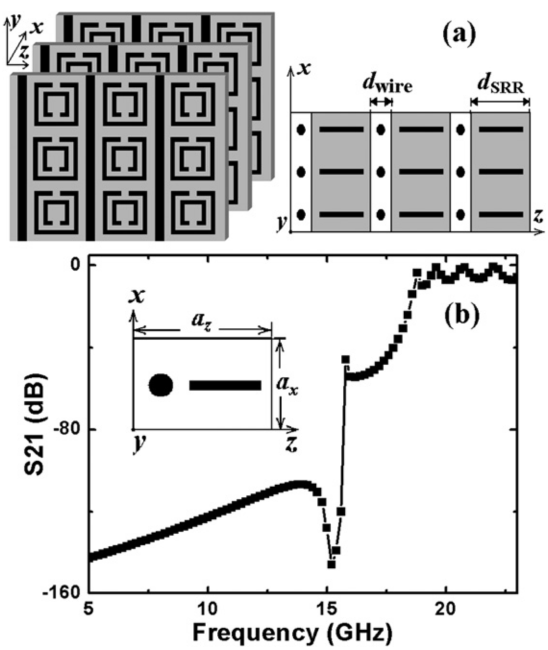

Fig. 5. (a) The in-plane configuration. The right panel is schematically shown to reveal the lamellar analogy, with wires and SRRs denoted by black dots and bars, respectively. (b) Transmission coefficient calculated with the unit cell in dimensions $a_x = 0.33$ cm, $a_y = 0.33$ cm, and $a_z = 0.443$ cm.

to the asymmetric structure in Ref. [20]). As far as the transmission coefficients are concerned (Fig. 4), there is no obvious difference from the off-plane case, which is easy to understand since shifting wires altogether along the z-axis does not influence the wires as a layer. This case also implies that electric response of wires and magnetic response of the SRRs do not obviously interact with each other. Accordingly, the wires are not necessary to be fabricated in the mirror planes of the SRRs for symmetry consideration; despite that nearly all of the SRR-wire metamaterials were intentionally prepared in this way [3,18,19].

3.3. In-plane case

For convenience during the fabrication process of SRR-wire metamaterial, Katsarakis et al. modified the generally adopted off-plane configuration into an in-plane case [Fig. 5(a)], in which configuration the SRRs and wires were positioned on the same side of the substrate [14,21,22]. Since this modification does not violate the original criterion that SRR-wire metamaterials have negative ε_{eff} of the wire array and negative μ_{eff} of the SRR array, it seems reasonable that the in-plane case should be LHM in the same way as an off-plane configuration. However, in the viewpoint of the ENG–MNG model, both the wire and SRR layers of the in-plane case are in the xy-plane (perpendicular to the propagating direction). Therefore, this case is different from the off-plane configuration, where the wire and SRR layers are in the yz-plane (parallel to the propagating direction). As a result, the ε_{eff} and μ_{eff} of the in-plane case can be written as follows:

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_{\text{SRR}} d_{\text{SRR}} + \varepsilon_{\text{wire}} d_{\text{wire}}}{d_{\text{SRR}} + d_{\text{wire}}} = \frac{\varepsilon_{\text{SRR}} d_{\text{SRR}}/d_{\text{wire}} + \varepsilon_{\text{wire}}}{d_{\text{SRR}}/d_{\text{wire}} + 1}, \tag{7}$$

$$\mu_{\text{eff}} = \frac{\mu_{\text{SRR}} d_{\text{SRR}} + \mu_{\text{wire}} d_{\text{wire}}}{d_{\text{SRR}} + d_{\text{wire}}} = \frac{\mu_{\text{SRR}} d_{\text{SRR}}/d_{\text{wire}} + \mu_{\text{wire}}}{d_{\text{SRR}}/d_{\text{wire}} + 1}. \tag{8}$$

To compare with the off-plane case, the formulas for ε_{eff} are kept in the same, while the formulas for μ_{eff} are changed. Therefore, whether or not the in-plane case is LHM depends on the sign of μ_{eff}. From Eq. (8), it is obvious that $\mu_{\text{eff}} < 0$ is satisfied only if $|\mu_{\text{SRR}}| d_{\text{SRR}} > \mu_{\text{wire}} d_{\text{wire}}$. That is, μ_{SRR} should be negative to a sufficiently large value in order that the in-plane configuration is a case of LHM. However, this is controversial to the fact that the negative μ_{SRR} is slightly negative in value, as has been specified in the off-plane case. Consequently, it is expected that the in-plane case under study would show no LH transmission peak. This is confirmed by the result in Fig. 5(b), from which no LH peak is observed within the resonant band of the SRRs. However, we emphasized that this conclusion is not necessary for all in-plane configurations because negative μ_{eff} could be obtained either through certain negative μ_{SRR} in large value for the resonant characteristic, or by controlling the ratio $d_{\text{SRR}}/d_{\text{wire}}$.

3.4. Thicker-SRR-layer case

As far as the off-plane case is concerned, it is found that, according the ENG–MNG interpretation represented by Eqs. (5) and (6), the values of ε_{eff} and μ_{eff} are not generally simultaneously negative for all $d_{\text{SRR}}/d_{\text{wire}}$ ratios. For example, increasing the ratio $d_{\text{SRR}}/d_{\text{wire}}$ sufficiently [Fig. 6(a)] will lead to positive ε_{eff} within the resonant band of the SRRs while negative μ_{eff} is kept in the same band. Consequently, the LH transmission peak should be suppressed and hence no LHM would be obtained. Consider the thicker-SRR-layer case shown in Fig. 6(a), the corresponding result shown in Fig. 6(b) indicates that this configuration does not form an LHM because the electric plasma frequency of this case (roughly 11 GHz) is much lower than the magnetically resonant frequency of the SRRs, and hence there is a stop band corresponding to the resonant regime of the SRRs, where otherwise should be a transmission peak if it is an LHM. For simplicity, the electric plasma frequency shift of the coupled system can be understood in a simple picture described as follows: In addition to that the split-ring resonators (SRRs) exhibits a magnetic resonance, it has electric response in the form of Drude dispersion which essentially resembles a cut-wire system. That is, the SRR-based structure has negative ε_{SRR} in the regime from the electric resonance frequency (ω_{eo}) to the electric plasma frequency (ω_{ep}), corresponding to the simulated results of about 27 GHz and 48 GHz, respectively (these two frequencies are confirmed in our simulations). Consequently, for the frequencies below ω_{eo} (27 GHz), the positive ε_{SRR} of the SRRs will shift the electric plasma frequency of the compound system down to a frequency lower than that of the wire-only system when the SRRs and the wires are coupled to be an effective medium.

On the other hand, this thicker-SRR-layer configuration might be wrongly understood as LHM according to the original criterion that SRR-wire metamaterials are LHM with negative ε_{eff} of

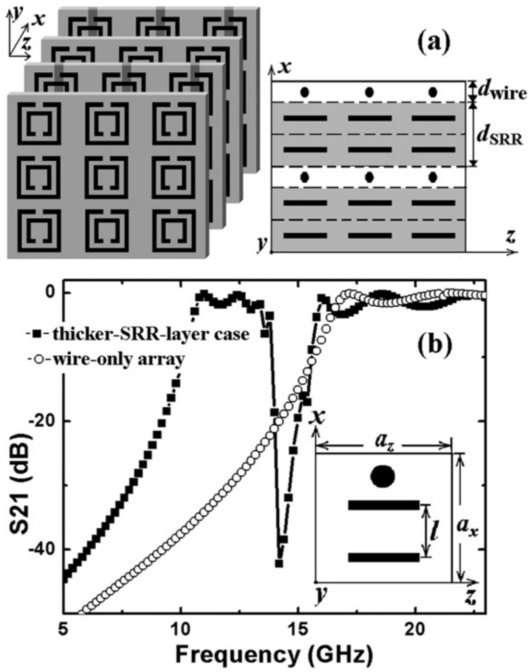

Fig. 6. (a) The thicker-SRR-layer configuration. The right panel is schematically shown to reveal the lamellar analogy, with wires and SRRs denoted by black dots and bars, respectively. (b) Transmission coefficient calculated with the unit cell in dimensions $a_x = 0.50$ cm, $a_y = 0.33$ cm, $a_z = 0.443$ cm, and $l = 0.25$ cm.

the wire array and negative μ_{eff} of the SRR array, without taking into account the fact that SRR-wire LHM is subjected to the ratio $d_{\text{SRR}}/d_{\text{wire}}$. In our simulations, the wire-only array (i.e., with SRR component removed away from the thicker-SRR-layer configuration) exhibits its electric plasma frequency at about 17 GHz [Fig. 6(b)], which implies that negative ε_{eff} of wire-only array should be expected within the resonant band of the SRRs. Therefore, this thicker-SRR-layer configuration might be wrongly understood as LHM if the original criterion is followed. As a matter of fact, SRR-wire metamaterials with lower electric plasma frequencies than their corresponding wire-only arrays were also experimentally noticed by Katsarakis et al. [14,23,24].

4. Conclusions

The ENG–MNG model is introduced to analogously interpret the SRR-wire LHM with numerical supports from different SRR-wire configurations, where the simulation results are consistent with the experimental phenomena (e.g., Refs. [14,23,24]). Additional remarks are summarized as follows:

The electric responses, or magnetic responses, are interacted to each other between the SRRs and wires. Concretely speaking, taking the off-plane case for example, the electric plasma frequency of SRR-wire metamaterial can be *much* lower than that of its wire-only array [see Eq. (5)], while the magnetic response of SRR-wire metamaterials contributed to the LH transmission peak is restricted to those negative μ_{eff} with small values [see Eq. (6)], that is why an LH peak might be found experimentally “moved” to *slightly* higher frequency than the magnetic resonant frequency of the SRRs [14] (it should be intuitively clear from retrieved results that negative μ_{eff} in small values are only located higher than and next to the resonant frequency of the SRRs). On the other hand, the electric response of wire array and the magnetic response of SRR array are not obviously coupled (i.e., negligible bianisotropic effect between the SRRs and the wires), which is true even in an asymmetric arrangement [20] (i.e., the wire-shifting configuration).

Acknowledgements

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