Light slowing down in Moiré fiber gratings and its implications for nonlinear optics

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A theory of the dispersion in the Moiré gratings is developed and it is shown that the group velocity of light in them can be reduced by up to three orders of magnitude. A conceptual similarity between Moiré grating and the electro-magnetic-induced transparency medium is demonstrated, and it is argued that for some applications the Moiré gratings present a simple viable alternative to electromagnetically induced transparency.

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The subject of light propagation in highly dispersive structures with a very slow group velocity (GV) has gained a lot of attention recently, thanks to the spectacular observation of the ultraslow (17 m/s) light propagation in cold Na vapor by Hau *et al.* [1] and later exciting results with the room-temperature Rb [2,3]. Although the idea of using the decrease in group velocity of light in the vicinity of the sharp resonance is an obvious one, only recently it could become practical due to the pioneering work of Harris and Yamamoto [4,5] on the electromagnetically induced transparency (EIT) that had provided the medium with extremely sharp resonances where high dispersion could be combined with very low absorption. Furthermore, since the nature of quantum interference in the EIT medium is such that only linear absorption is being canceled, while the nonlinear susceptibility is not, a number of proposals have been made for using the EIT medium in the low-light-level nonlinear optics [6]. More recently, EIT in photonic gap structures have been considered in [7].

At the same time, recent years have seen the most rapid progress in the development of new optical medium with engineered dispersion—the fiber Bragg gratings [8–10], where, in the vicinity of the photonic band gap the considerable slowing down of light is also feasible. Nonlinear phenomena in Bragg gratings, including soliton propagation have been studied in a number of works [11,12]. To avoid the problems associated with high GV dispersion (GVD), the use of two cascaded gratings had been suggested [13] and experimentally demonstrated [14] when the light slowing down by a factor of 1.5 is observed. Further reduction of the GV would require very narrow spacing between the stop bands of two gratings and thus would be extremely difficult to control. If, however, one uses a ''superstructure'' grating [15–17], where two gratings are superimposed, rather than cascaded, the situation is different—whatever is the spacing between the Bragg wavelengths of two superimposed gratings, there will always be a narrow transmission band between them. The superstructure, or Moiré fiber gratings (MFG), were first proposed as narrow transmission filters [15,18] and are being used in the tunable lasers. More recently, their dispersive properties in reflection had attracted attention [19], and nonlinear propagation [20] and localization [21] effects in some superstructure gratings have been observed.

To the best of our knowledge, the dispersion properties of the MFG in transmission mode had not been analyzed since most studies treated only the reflectivity and transmission [8,10]. The difficulty lies in the fact that two coupled Bloch waves [9] cannot adequately describe the MFG, since, as is shown below, the number of coupled waves propagating in it is infinite. In this paper, we rigorously analyze the dispersion in the MFG's and show that GV of light can be slowed down by a factor of up to 1000, which, combined with small GVD, makes them attractive candidates for true time delay lines in photonic systems and for the GVD compensation in communication links. We also show that MFG is conceptually analogous to the EIT system, and, given the many-fold enhancement of electric field in the MFG, the effective nonlinearities in it can be increased by up to three orders of magnitude, making it a viable alternative to EIT for low-light-level nonlinear optics.

To start, the effective index of refraction of the MFG can be described as

$$n(x)=\bar{n}+\delta n\cos\frac{2\pi}{\Lambda_s}z\cos\frac{2\pi}{\Lambda}z, \tag{1}$$

where Λ is the Bragg period, and Λ_s is a superstructure, or Moiré period. Introducing the corresponding spatial frequencies, $g=2\pi/\Lambda$, and $G=2\pi/\Lambda_s$ one can rewrite Eq. (1) as

$$n(x)=\bar{n}+\frac{1}{2}\delta n\cos(g+G)z+\frac{1}{2}\delta n\cos(g-G)z. \tag{2}$$

According to [9], Bragg reflection should cause the appearance of two stopbands of width $\Delta\omega_{stop}=\omega\delta n/\bar{n}$ in the transmission spectrum, centered at $\omega_1=(c/2\bar{n})(g-G)$ and $\omega_2=(c/2\bar{n})(g+G)$, respectively, as shown in Fig 1(a). This simple picture holds only for as long as the allowed (high-transmission) band between two gaps is much wider than the gaps themselves. When this is no longer true, the interaction becomes more complex—the wave reflected back by the first grating $(g-G)$ gets redirected back by the second grating $(g+G)$, only to be scattered back, and so on. Thus the eigenmode of MFG consists not of just two plane waves but of an infinite number of backward and forward waves. Two stopbands ''repel each other,'' resulting in a formation of the allowed band no matter how close ω_1 and ω_2 are. The dispersion curve in the allowed band becomes flattened, as shown in Fig. 1(b), resulting in a decrease of GV. To evaluate the dispersion curve, we extend the approach of [9] to

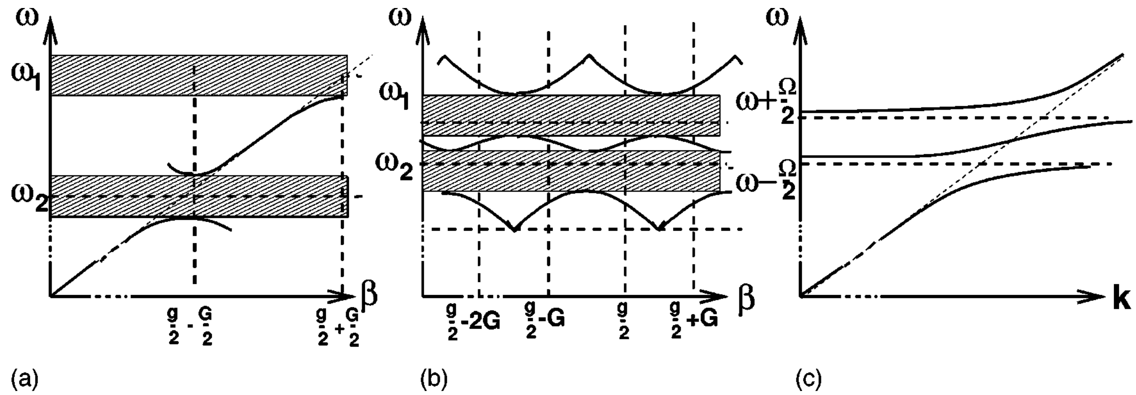

FIG. 1. Dispersion curves for (a) two widely spaced gratings, (b) MFG with narrow passband, (c) EIT medium.

represent the wave propagating in the MFG as a superposition of an infinite number of plane waves

$$E(z,t)=\sum_m \sum_n A_{mn}\exp\{i[\beta+m(g+G)+n(g-G)]z-i\omega t\}+\text{c.c.},\tag{3}$$

where $m,n=0,\pm 1,\pm 2,\ldots$. Now, we are interested in the frequencies that are contained between two first-order stopbands, in the vicinity of the ''central frequency'' $\omega_0=cg/2\bar{n}$. For such frequencies, only the nearly resonant forward $m+n=0$ and backward $m+n=-1$ waves will be strongly represented in the expansion (3). Introducing the new index $k=m-n$ we obtain

$$E(z,t)=\left[\sum_{k=2m} A_k\exp\{i(\beta+kG)z\}+\sum_{k=2m+1} A_k\exp\{i(\beta+kG-g)z\}\right]\exp\{-i\omega t\},\tag{4}$$

where all the terms with even $|k|$ propagate forward and the terms with odd $|k|$ propagate backward. The one-dimensional Helmholtz equation is then

$$\frac{d^2E}{dz^2}-\overline{n^2}\frac{\omega^2}{c^2}E=\frac{\omega^2}{c^2}\bar{n}\,\delta n\cos(g+G)z+\frac{\omega^2}{c^2}\bar{n}\frac{1}{2}\,\delta n\cos(g-G)z.\tag{5}$$

Substitute Eq. (4) into Eq. (5), multiply by a complex-conjugate term $\exp\{-i(\beta+kG)z\}$ for even k or $\exp\{-i(\beta+kG-g)z\}$ for the odd k and integrate over dz to obtain

$$\left[(\beta+kG)^2-\overline{n^2}\frac{\omega^2}{c^2}\right]A_k=\frac{\omega^2}{c^2}\bar{n}\,\delta n(A_{k-1}+A_{k+1}),\quad k=2m,\tag{6}$$

$$\left[(\beta+kG-g)^2-\overline{n^2}\frac{\omega^2}{c^2}\right]A_k=\frac{\omega^2}{c^2}\bar{n}\,\delta n(A_{k-1}+A_{k+1}),\quad k=2m-1.\tag{7}$$

Now certain simplifications can be made. Introduce small deviations of wave vector $\delta\beta=\beta-g/2\ll g/2$ and frequency $\delta\omega=\omega-\omega_0\ll\omega_0$, and then normalize all the variables to the modulation depth of index grating $\delta n/(2\bar{n})$ by introducing $\tilde{\omega}=2\bar{n}\delta\omega/\delta n\omega_0$, $\tilde{\beta}=2c\delta\beta/\delta n\omega_0$, and $\tilde{G}=2cG/\delta n\omega_0$ to obtain the final homogeneous system of equations

$$[\tilde{\beta}-(-1)^m(\tilde{\omega}+k\tilde{G})]A_k=(-1)^m(A_{k-1}+A_{k+1}).\tag{8}$$

The characteristic matrix for Eq. (8) has an infinite number of rows and columns, but, since only three terms are present in each, the diagonalization is easy to accomplish numerically, by truncating the number of plane waves at some k_{max} and then solving the characteristic equation recursively to obtain the dispersion curve $\tilde{\omega}(\tilde{\beta})$. In our calculations, depending on the value of $\tilde{G}$, k_{max} varied from 10 to 50. A typical result is shown in curve *a* in Fig. 2 for $\tilde{G}=0.6$, assuming the 0.1% modulation of index and $\lambda_0=1.55\ \mu$m corresponds to $\Lambda_s\approx 3.3$ mm. As expected, a very narrow transmission band has been opened. The full width of the

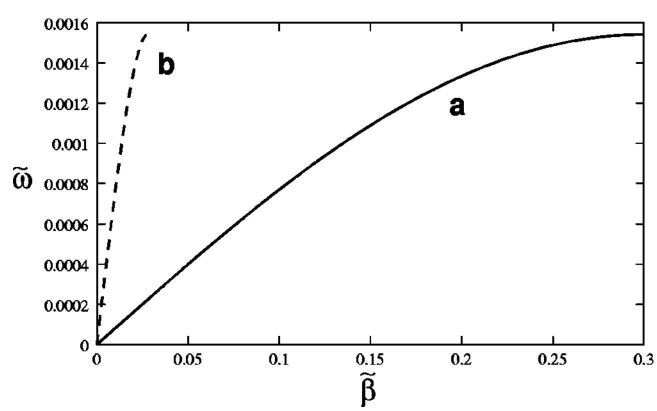

FIG. 2. Dispersion curves in two fiber gratings with identical 320 MHz passband: *a*, MFG with $\lambda_0=1.55\ \mu$m, $\Lambda_s=3.3$ mm, *b*, two cascaded gratings with $\Lambda_1=5161$ Å and $\Lambda_2=5172$ Å .

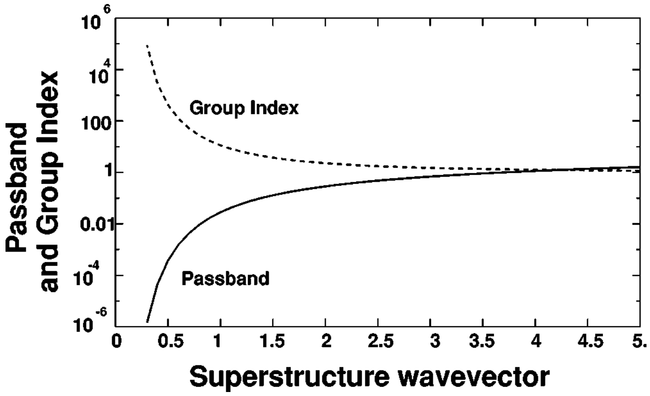

FIG. 3. Passband width and group index of MFG as functions of normalized superstructure wave vector $\tilde{G}$.

transmission band is $\Delta\tilde{\omega}=3.2\times10^{-3}$, i.e., $\Delta\nu\approx320$ MHz ($\Delta\lambda$ 1.6×10^{-2} Å). The GV $v_{gr}=\partial\omega/\partial\beta=(c/\bar{n})\times(\partial\tilde{\omega}/\partial\tilde{\beta})$ is slowed down by a factor of 125.

For comparison, the curve *b* shows the dispersion of two cascaded gratings of different periods, $\Lambda_1=5161$ Å and $\Lambda_2=5172$ Å with the same index modulation $\delta n=0.1\%$. Although the same narrow passband exists, the GV is decreased only by a factor 25. Therefore, MFG's have a clear advantage over the cascaded gratings when it comes to slowing down the light. Furthermore, to assure the existence of the narrow passband in the cascaded gratings mentioned here, the periods of these gratings needs to be maintained with the precision better than the width of the passband, i.e., of the order of 10^{-2} Å, which appears extremely difficult. On the other hand, with MFG, small deviations of the main grating period Λ only shift the the center of the passband, while small deviations in the superstructure period Λ_s can slightly affect the width of the passband, but not its existence.

The dependences of the width of the passband $\Delta\tilde{\omega}$ and the relative group index $n_{gr}=(c/n)\partial\beta/\partial\omega=c(\partial\tilde{\beta})/\partial\tilde{\omega}$ on the superstructure wave vector $\tilde{G}$ are shown in Fig. 3. As expected, the significant slowing down starts taking place when the distance between two forbidden gaps becomes first comparable, and then smaller than the full width of the forbidden band, i.e., when $\tilde{G}\leqslant2$. The group index and passband width follow a rather simple empirical formula $n_{gr}\approx0.33\tilde{G}/\Delta\tilde{\omega}$ through the large range of $\tilde{G}$. The important question to be asked is how large can the group index be made? A simple estimate can be made from the following considerations: variation of the central frequency ω_0 should not exceed the width of the passband $\Delta\omega=\Delta\tilde{\omega}\delta n\omega_0/\bar{n}$. But from the uncertainty principle, in the waveguide of length L, $\delta\omega_0\sim2\pi c/\bar{n}L$. Thus we obtain

$$\Delta\tilde{\omega}_{min}\approx\frac{2}{\delta n}\frac{\lambda}{L},\quad n_{gr}^{max}\approx0.17\tilde{G}_{min}\delta n\frac{L}{\lambda},\tag{9}$$

where $\tilde{G}_{min}$ is always of the order 1. For the 1.5-m-long grating with 0.1% index modulation, one can expect $\Delta\tilde{\omega}_{min}\sim2\times10^{-3}$ and $n_{gr}^{max}\sim150$, i.e., the results in Fig. 2 are quite close to the limit imposed by the length.

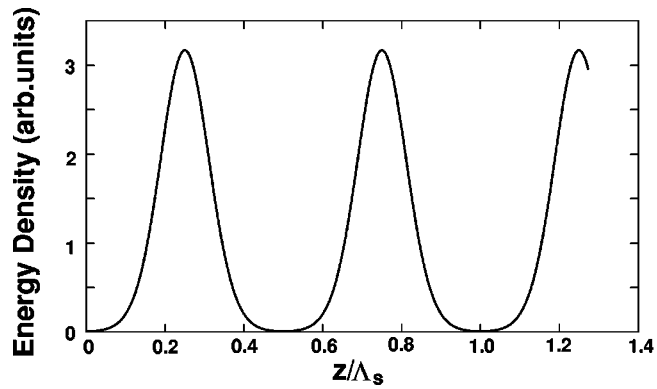

FIG. 4. Light distribution inside MFG for the example of Fig. 2(a).

Consider now the distribution of the electromagnetic energy in the MFG with the parameters previously described (Fig. 4). A number of spatial harmonics of G (about four in the example shown here) in the propagating wave results in the energy distribution in space that looks quite similar to the energy distribution in time in the ultrashort mode-locked pulse train [22]. The energy density is maximum at $z/\Lambda=m+1/2$, i.e. where the index modulation is minimal—this fact reveals a well-known fact that MFG is effectively a succession of coupled Fabry-Perot cavities.

Let us now compare the results obtained here with the EIT experiments [1,6,4]. As shown in Fig. 1(c), the reduction of the GV of light in EIT medium follows from the fact that the dispersion curve is constrained between two dressed-state resonances with frequencies $\omega_0\pm\Omega_c/2$, where Ω_c is the Rabi frequency of the coupling field between two states with relatively broad (γ_b) and very narrow (γ_n) linewidths. EIT can be observed when the condition $\Omega_c^2\gg\gamma_b\gamma_n$ is satisfied. Using the assumption that all the lines have natural linewidth, it is not difficult to obtain from Eq. (2c) in [6] the expression for the maximum achievable GV reduction:

$$n_{gr,EIT}^{max}\sim\frac{1}{2\pi^2}N\lambda^3\frac{\omega}{\gamma_n}.\tag{10}$$

Comparison of Eq. (10) with Eq. (9) confirms the analogy between EIT and MFG, also evident from Figs. 1(b) and 1(c). The GV reduction in both cases results from the strong coupling - with atomic oscillations in EIT and with the counterpropagating waves in MFG. The magnitude of reduction is determined by the strength of that interaction (density of atoms in EIT vs index modulation in grating) multiplied by the characteristic Q factor of the system (ω/γ in EIT vs L/λ in MFG). Furthermore, unless one deals with Bose condensate [1], in order to avoid broadening it is necessary to keep $N\lambda^3\ll1$, just as in order to avoid scattering it is necessary to keep $\delta n\ll1$. Therefore, it can be said that the relative merits of two methods are determined by their Q's. Since the Q of the atomic line can be extremely high, the EIT method allows the GV reduction up five orders of magnitude using cold atoms. With MFG the reduction is far more modest, two-to-three orders of magnitude in long (10 m) fibers. Yet fibers do offer a number of obvious advantages—simplicity, low cost, no maintenance, and ability to be designed and tuned for any specific wavelengths. Furthermore, the bandwidth of MFG is higher. For a number of applications, such as optical delay lines, the two-to-three order reduction in speed is sufficient and the MFG can be successfully used there.

We can now turn our attention to the nonlinear optical processes in EIT media [5,6] and MFG. In EIT media the energy density is increased by a factor $n_{gr,EIT}$, but the field strength remains the same as outside of it—the energy is simply stored in atomic polarization oscillations. The increase in nonlinear susceptibility comes from the fact that the effective detuning can be made as small as $\Omega_c \sim (\gamma_b \gamma_n)^{1/2}$, vs γ_b in normal nonlinear materials. For a large number of third-order processes, the enhancement in $\chi^{(3)}$ is then γ_b/γ_n. When one assumes that γ_b is the natural linewidth of an atomic transition, say 0.1 GHz, and $\gamma_n \sim 100$ Hz, the enhancement can be as high as six orders of magnitude.

In the MFG, the nonlinear susceptibility itself is not increased at all, but the average electric field strength does increase inside the medium, $E_{in}^2 \sim n_{gr} E_{out}^2$. Furthermore, the ''bunching'' of field (Fig. 4) provides additional increase in electric field. For the typical $\chi^{(3)}$ process the enhancement can be as high as three orders of magnitude. Considering much longer length of fiber, the actual efficiency of the process can be higher than in EIT. The EIT scheme does have one substantial advantage over the MFG—since the index of refraction is close to one, no phase matching is required. For this reason, the applications of MFG's for nonlinear optics are limited to those where the frequency conversion does not take place—Kerr nonlinearity, two-photon absorption, and Brillouin scattering.

In conclusion, we developed, for the first time, to the best of our knowledge, the theory of dispersion in the MFG and had shown that the reduction of the speed of light in them can be as high as two-to-three orders of magnitude. We have pointed out an interesting analogy between MFG and the EIT, and had shown that MFG gratings exhibit linear and nonlinear optical properties similar to the EIT medium, and, for a number of applications MFG can present a simple and reliable alternative to EIT.

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