



Enhancement and manipulation of group delay based on topological edge state in one-dimensional photonic crystal with graphene

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Abstract: In this paper, the reflected and transmitted group delay from a one-dimensional photonic crystal heterostructure with graphene at communication band are investigated theoretically. It is shown that the negative reflected group delay of the beam in this structure can be significantly enhanced and can be switched to positive. The large reflected group delay originates from the sharp phase change caused by the excitation of topological edge state at the interface between the two one-dimensional photonic crystals. Besides, the introduction of graphene provides an effective approach for the dynamic control of the group delay. It is clear that the positive and negative group delay can be actively manipulated through the Fermi energy and the relaxation time of the graphene. In addition, we also investigate the transmitted group delay of the structure, which is much less than the reflected one. The enhanced and tunable delay scheme is promising for fabricating optical delay devices like optical buffer, all-optical delays and other applications at optical communication band.

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1. Introduction

The delay effect of optical transmission is an abnormal physical phenomenon in devices and media with high dispersion. By controlling the dispersive properties of medium, delay time of reflected and transmitted pulses enables transmission and manipulation from subluminal to superluminal [1,2]. The interaction between light and matter can be well understood through the study of optical group delay, so as to promote the application in many fields such as optical buffer [3], tunable all-optical delays [4], all-optical switching [5], all-optical memory [6] and so on. Therefore, the group delay phenomenon has been extensively studied in various structures in recent years, such as Otto structure [7], left-handed medium [8], weakly absorbing dielectric slab [9], and photonic crystal waveguides [10]. Recently, superluminal/subluminal group delay phenomena in structures like molecular-aggregate nanofilms [11], plasmon-induced transparency structures [12], and isotropic-anisotropic photonic crystals [13] have also been reported. Researchers realized that it was very crucial and compelling that the group delay be flexibly controlled in the transmission process of optical pulse in solid medium [14,15]. In recent years, graphene has attracted much attention in the field of optical group delay as a two-dimensional material with excellent photoelectric properties such as support for exciting surface plasmons polaritons (SPPs) [16], broadband [17], etc. It not only shows an unprecedented ability to enhance the interaction between light and matter, but also provides a good condition for controlling group delay [18–20]. The light pulse propagation is mainly manipulated by controlling the electrical conductivity of graphene in various graphene-based systems [21,22]. In particular, graphene was employed to slow down and control the speed of light in photonic crystal waveguide with air holes [23]. Tunable plasmonically induced transparency window with a group delay by

graphene ribbon waveguide was achieved [24]. Tunable positive and negative group delays of optical pulse reflection in a graphene-based layered system related to resonances and surface plasmon are proved [25]. With the development of communication, computer simulation and micro nano processing technology, the optical group delay has been widely studied based on various micro-nano structures. However, it is still a challenge to realize optical delay devices with simpler structure, more controllable performance and large delay value in communication band.

The introduction of topological states has completely changed our understanding of electromagnetic wave propagation and scattering. During the last few years, optical topological states have attracted extensive attention due to the unique characteristics like uni-directional transport edge state and topological protection [26], and topological states in different optical structures have been successively discovered and confirmed [27–29]. It has been widely studied in Fano-resonance [30], quantum spin Hall states [31], reconfigurable electromagnetic pathways [32], optical delay lines [27] and etc. In addition, optical topological states usually exist at the interface of one-dimensional photonic crystal (PhC) [33,30] and can generate transmission peaks in the whole heterostructure, which means that the electric field is highly localized at this interface [34], and the reflected phase will vary greatly at a specific frequency. This characteristic of optical topological states phenomenon provides the possibility to realize large group delay devices at optical communication band by using the photonic crystal heterostructure.

In this paper, we theoretically demonstrated that large group delay can be achieved at communication band by exciting the optical topological state through the PhC composite structure. The subluminal and superluminal effects can be dynamically controlled by adjusting the parameters of graphene. The large tunable optical group delay at communication band can be applied in the fields of subluminal and superluminal control, photoelectric detection, communication sensing and so on.

2. Theoretical model and method

We consider a graphene one-dimensional PhC composite structure to excite the topological state, which is composed of PhC1, PhC2 and graphene, as shown in Fig. 1. PhC1 and PhC2 are alternately formed by A (Si) and B (SiO₂) with a period of 4, the refractive index is set to be 1.46 for A, 2.82 for B, and their permittivity are ϵ_A and ϵ_B , respectively. Among them, for the thickness of the A and B, we set $d_{A1} = 815$ nm, $d_{B1} = 680$ nm in PhC1, and $d_{A2} = 1290$ nm, $d_{B2} = 685$ nm in PhC2. The incident angle we set: $\theta = 0$. It is well known that graphene is a two-dimensional material with a single atomic layer. We usually use surface conductivity to characterize its photoelectric properties. Without considering the external magnetic field and under the random-phase approximation, the surface conductivity of graphene σ_0 is the sum of the interband σ_{inter} and the intraband σ_{intra} , where [35]:

$$\sigma_0 = \sigma_{intra} + \sigma_{inter}, \quad (1)$$

$$\sigma_{intra} = \frac{ie^2k_B T}{\pi h^2(\omega + i/\tau)} \left(\frac{E_F}{k_B T} + 2 \ln \left(e^{-\frac{E_F}{k_B T}} + 1 \right) \right), \quad (2)$$

$$\sigma_{inter} = \frac{ie^2}{4\pi\hbar} \ln \left| \frac{2E_F - (\omega + i\tau^{-1})\hbar}{2E_F + (\omega + i\tau^{-1})\hbar} \right|, \quad (3)$$

where e is electron charge and ω is angular frequency, τ is the relaxation time, \hbar is reduced Planck's constant. E_F is the Fermi energy of graphene and $E_F = \hbar v_F \sqrt{\pi n_{2D}}$, where $v_F \approx 10^6$ m/s is the Fermi velocity of electrons, n_{2D} is the carrier density. The carrier density can be controlled by electrostatic doping with external gate voltage, which affects the Fermi energy of graphene [36]. Here, we set $N = 1$, $T = 300$ K, $\tau = 0.5$ ps and $E_F = 0.53$ eV. The parameters are consistent in the following paper except for special instructions.

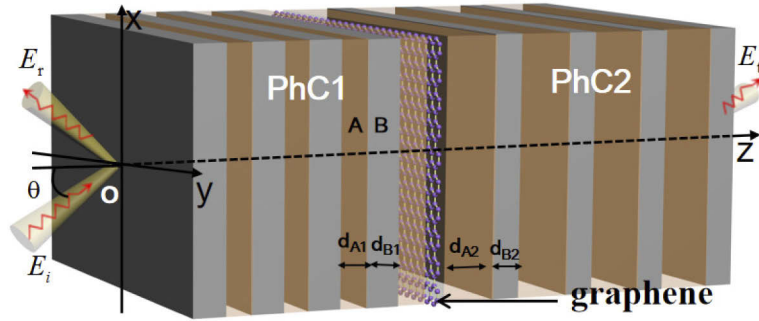


Fig. 1. Schematic diagram of one-dimensional photonic crystal structure with graphene.

As we all know, the transfer matrix method is a classical method to investigate one-dimensional PhC. When the electromagnetic wave is transmitted in the uniform single dielectric layer, the propagation matrix can be expressed as:

$$P(d) = \begin{bmatrix} e^{-ik_z d} & 0 \\ 0 & e^{ik_z d} \end{bmatrix}. \quad (4)$$

The relationship between the electromagnetic fields on both sides of the interface can be obtained from the electromagnetic field boundary conditions. For TE polarization, the transmission matrix can be expressed as:

$$D_{1 \rightarrow 2} = \frac{1}{2} \begin{bmatrix} 1 + \eta & 1 - \eta \\ 1 - \eta & 1 + \eta \end{bmatrix}, \quad (5)$$

where $\eta = k_{2z}/k_{1z}$, and k_{jz} ($j = 1, 2$) is the wave vector of light propagating in dielectric 1 and dielectric 2. In this paper, the conductivity of graphene can be reflected in the boundary conditions [37] as graphene is only one atom thick:

$$D_{1 \rightarrow 2} = \frac{1}{2} \begin{bmatrix} 1 + \eta + \xi & 1 - \eta + \xi \\ 1 - \eta - \xi & 1 + \eta - \xi \end{bmatrix}, \quad (6)$$

where $\xi = \sigma_0 \mu_0 \omega / k_{1z}$. Obviously, ξ is highly dependent on the conductivity of graphene, which provides an effective way for the dynamic regulation of group delay in proposed structure.

Based on transfer matrix method, the transmission matrix of the entire system can be easily obtained and the reflection coefficient (transmission coefficient) and reflection phase (transmission phase) are further got. The group delay can be expressed as follows assuming the incident pulse is a Gaussian pulse [38,27]:

$$\tau_r = [\partial \phi_r / \partial \omega]_{\omega = \omega_c}, \quad (7)$$

where ϕ_r is the reflected phase and ω_c is the carrier frequency. The definition of transmitted group delay is similar to reflected group delay.

3. Results and discussions

First, we discuss the reflectance of Phc1, Phc2 and the composite structure of PhC with or without graphene, as shown in Fig. 2(a). It can clearly be seen that the optical topological state cannot be excited in single PhC, which has a large photonic band gap around 195THz, makes the reflectance close to 1. By constructing PhC heterojunction, the optical topological state is excited on the

contact surface of the PhC, and the reflectance drops to near zero. In our previous work [39], we have calculated the Zak phases and demonstrated that the topological states were excited in a similar structure. At the same time, we plot the electric field distribution by COMSOL Multiphysics, as shown in Fig. 2(b). It can be seen that the appearance of the topological state is accompanied by the phenomenon of local field enhancement. The reflectance of whole structure exhibits a sharp reflection peak within the band gap. This sharp reflection peak corresponds to the sharp change of the phase near the 195 THz. It can be seen from Eq. (7) that the sharp phase change makes it possible to obtain large group delay near the resonance point. At the same time, in order to dynamically adjust the group delay, monolayer graphene sheet is placed between the two photonic crystals. For convenience, it is assumed that graphene and dielectric surface are non-contact, so the influence of graphene on the group delay is mainly reflected in the transmission matrix of the contact surface of two photonic crystals. It can be concluded that small changes in the conductivity of graphene can achieve dynamic control of group delay. It can be seen from the short dash dot curve in Fig. 2(a) that the introduction of graphene can not only maintain the reflectivity dip at 195THz, but also provide us with an effective means to regulate group delay.

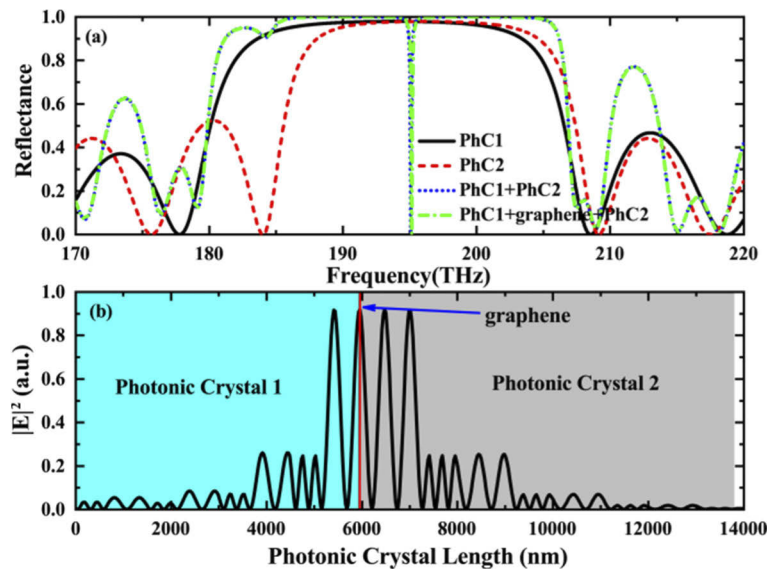


Fig. 2. (a) Dependence of reflectance on frequency in Phc1(solid line), Phc2(dashed line), composite structure without graphene (dotted line), composite structure with graphene (dash-dotted line). (b) Electric field distribution of composite structure (black solid line).

For simplicity, we assume that the electromagnetic wave incident vertically. It can be seen from Fig. 3 that the phase change of the PhC heterostructure is continuous and has a very large slope by exciting the Topological edge state, which results in a large reflected group delay of about when graphene is not introduced (solid line). In addition, according to Eq. (1)–(3) of the conductivity of graphene, the Fermi energy has a significant effect on the conductivity of graphene. According to the transfer matrix method, the reflectance is directly related to the conductivity, thus the reflected phase and reflected group delay are affected by the conductivity too. Therefore, the sign and value of group delay can be dynamically adjusted by controlling the external voltage to change the Fermi energy. As shown in Fig. 3, a group delay about -1134 ps can be obtained when the Fermi energy is 0.53 eV. If the Fermi energy continues to increase, a

group delay with a greater negative value can be obtained. For example, when the Fermi energy is 0.55 eV, the group delay is about -1854 ps. However,

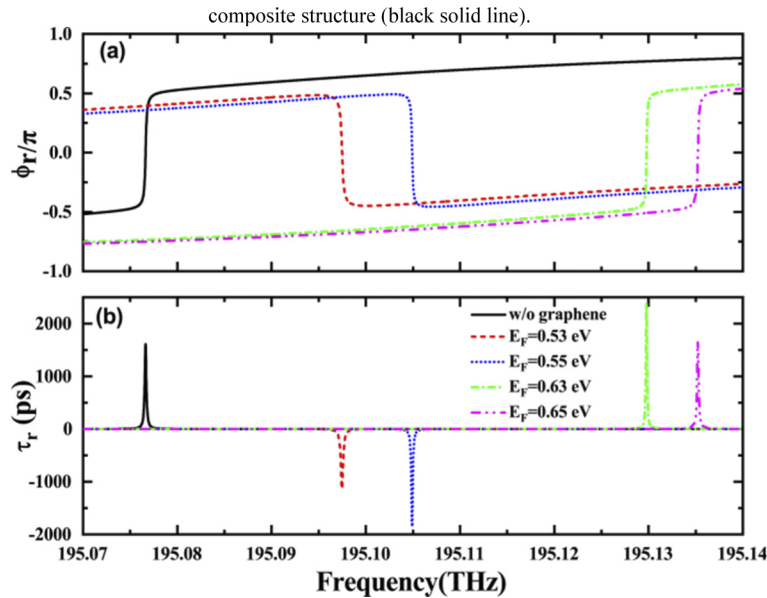


Fig. 3. Dependence of the (a) reflected phase ϕ_r , and (b) reflected group delay τ_r on frequency for different Fermi Energy of graphene. For comparison, reflected phase ϕ_r , and reflected group delay τ_r without graphene is shown as well (solid line).

when the Fermi energy increases to a certain value (0.58 eV), the slope of the reflected phase changes from negative to positive, resulting in the group delay suddenly changing from negative to positive, resulting in the group delay suddenly changing from negative to positive. Moreover, in a certain range after the jump, the group delay is even greater than that before the introduction of graphene. Taking 0.63 eV as an example, the group delay is about 2372 ps, 46.7% higher than that before the introduction of graphene. Such a large positive or negative group delay can be applied in photonic devices such as time delay lines and phase shifter [18]. At the same time, these phenomena indicate that the applied voltage plays an important role in determining the sign and value of group delay. The regulation of the Fermi energy by external voltage provides an effective way to control the reflected group delay by external control means in a fixed structure.

In addition, Eqs. (1)–(3) also indicate that the relaxation time has a significant effect on the conductivity of graphene, it can be inferred that the reflected group delay is also very sensitive to the relaxation time. On the premise that other parameters are the same as in Fig. 3, the effect of relaxation time on the reflected group delay is shown in Fig. 4 when Fermi energy is 0.53 eV. Compared with the Fermi energy, the increase of relaxation time will only lead to the increase of phase slope and reflected group delay, and will not change the central frequency of reflected phase and group delay. The reflected group delay is about -433 ps when $\tau = 0.3$ ps. With the increase of the relaxation time, the slope of the reflected phase and the group delay become larger and larger. The reflected group delay is about 1907 ps when $\tau = 0.6$ ps. Although the relaxation time is difficult to adjust for the fixed structure, for the graphene monolayer, it can not only control the reflected group delay by external voltage, but also affect the delay by changing its own parameters. This idea has unique advantages compared with the previous various means and ways to achieve delay. Considering the thickness of single-layer graphene is very thin (only 0.34 nm), the conductivity with few layers ($N < 6$) can be approximately expressed as $\sigma \approx N\sigma_0$. Under this assumption, it is convenient to discuss the influence of the number of graphene layers

on the reflected group delay, as shown in Fig. 5. The increase of graphene layer number is equivalent to the linear increase of surface conductivity of the medium, and the real part of the reflection coefficient is increased. Correspondingly, increasing the number of graphene layers can reduce the slope of the reflected phase and the reflected group delay. When the number of layer $N = 1$, the value of the reflected group delay is the largest, about -1134 ps, and when N is increased to 5, the value of the reflected group delay is reduced to about -143 ps. Obviously, the change of the layer number has a great influence on the group delay after the introduction of multilayer graphene. These phenomena clearly show that the layer number of graphene also plays an important role in determining the group delay.

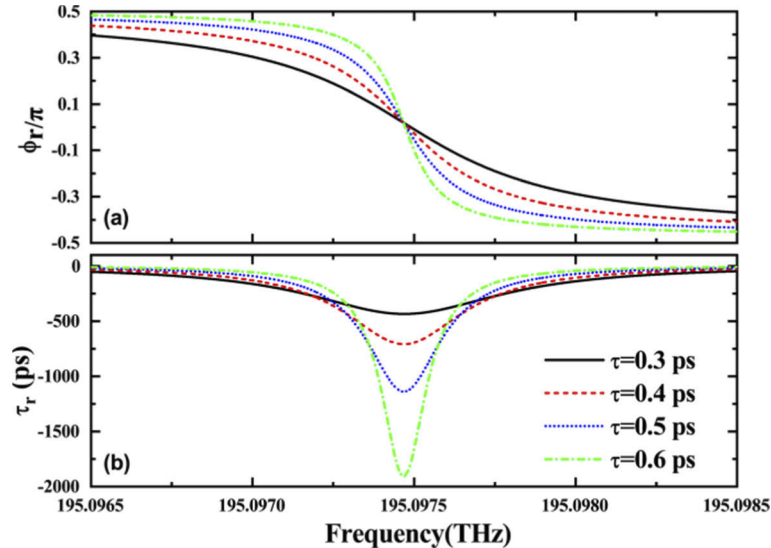


Fig. 4. Dependence of the (a) reflected phase ϕ_r , and (b) reflected group delay τ_r on frequency for different electron-phonon relaxation times τ of graphene.

Finally, we investigate the transmitted group delay based on the topological state of one-dimensional photonic crystal structure with graphene. On the premise that other parameters are the same as in Fig. 3, the effect of Fermi energy on the transmitted group delay is shown in Fig. 6. The dramatic phase change of reflected group delay does not occur in the transmitted one. We find that the reflected group delay is much larger than transmitted one. For example, the transmitted group delay of the structure without graphene (solid line) is about 3.06 ps while reflected one is 1616 ps. Although the optical topological state does not significantly amplify the transmitted group delay of this structure, it is very interesting to adjust the transmitted group delay after the introduction of graphene. We find that when the Fermi energy is 0.31 eV, the group delay is about 1.03 ps. However, when the Fermi energy is increased to 0.41 eV, the transmitted group delay increases gradually about 3 ps, but the increase amplitude is very small. For example, when the Fermi level is 0.41 eV, the group delay is 2.09 ps, and when the Fermi level is 0.51 eV, the group delay is 3.04 ps. Although the value of transmitted group delay is relatively small, the study of transmitted group delay based on optical topological state can provide another idea for the design of optical group delay devices in the future.

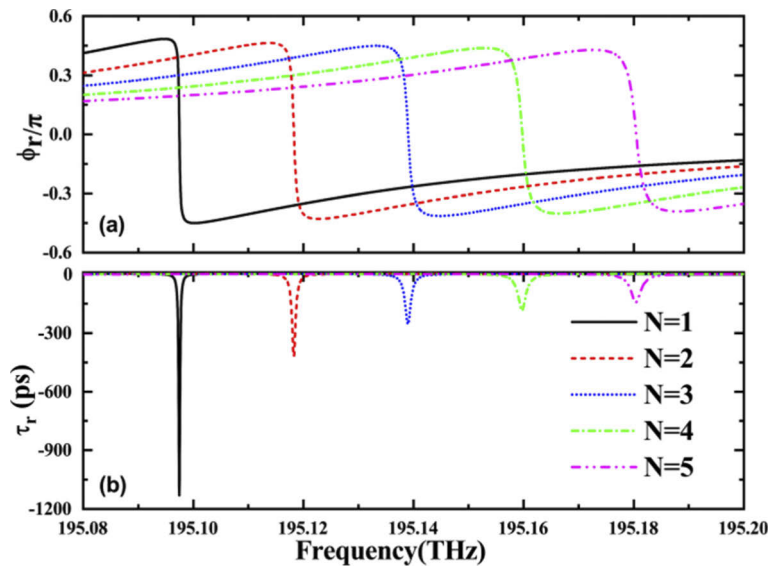


Fig. 5. Dependence of the (a) reflected phase ϕ_r , and (b) reflected group delay τ_r on frequency for different layers N of graphene.

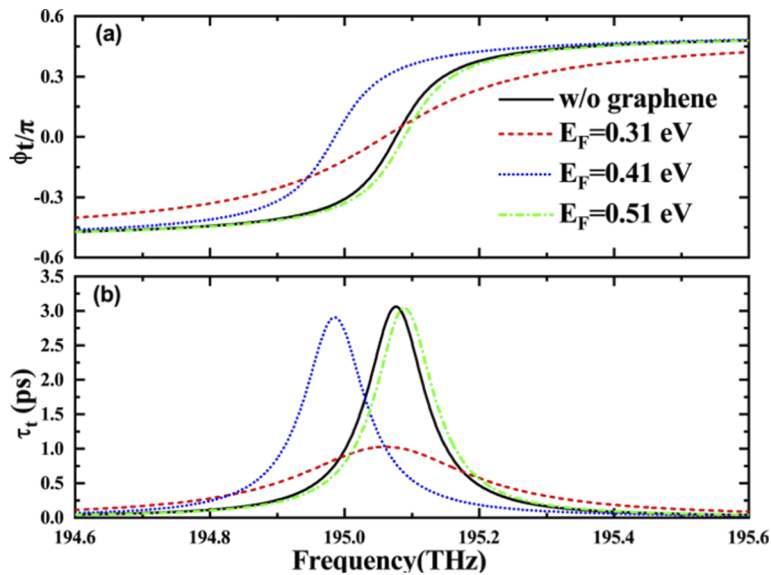


Fig. 6. Dependence of the (a) transmitted phase ϕ_t , and (b) transmitted group delay τ_t on frequency for different Fermi Energy of graphene. For comparison, transmitted phase ϕ_t , and transmitted group delay τ_t without graphene is shown as well (solid line).

4. Conclusion

In conclusion, the reflected and transmitted group delay based on the topological state of one-dimensional PhC composite structure with graphene at optical communication band has been investigated. The excitation of the topological state makes the reflectance appear a Sharp reflection peak in the band gap, and the reflected group delay in this structure is significantly enhanced. Moreover, not only the value of group delay can be changed, but also the conversion

and regulation between positive and negative reflected group delay can be realized by adjusting the Fermi energy of graphene. We further found that the reflected group delay is also affected by the relaxation time and the number of graphene layers. This means the reflected group delay depends to a great extent on the nature of graphene, so it provides a feasible way to enhance and control the reflected group delay. In addition, we also found that the transmitted group delay is also affected by graphene, while it is much less than reflected one. We believe that the large tunable group delay realized by excitation of topological states has potential applications to realize integrated micro/nano scale fast and slow light control, photoelectric detection and communication sensor devices in optical communication band.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but maybe obtained from the authors upon reasonable request.

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