

# Localization and oscillation of optical beams in Moiré lattices

RUI-DONG XUE,<sup>1,2</sup> WEI WANG,<sup>1,2</sup> LU-QI WANG,<sup>1,2</sup> HAO-LIN CHEN,<sup>1</sup>  
RUI-PENG GUO,<sup>1</sup> AND JING CHEN<sup>1,\*</sup>

<sup>1</sup>MOE Key Laboratory of Weak-Light Nonlinear Photonics, School of Physics, Nankai University, Tianjin 300071, China

<sup>2</sup>These authors contribute equally to this work

\*jchen4@nankai.edu.cn

**Abstract:** We study the propagation of optical beams in two-dimensional Moiré lattices, and demonstrate position-dependent beam dynamics when a quasi-Bragg condition is satisfied. We show that when the optical beam is incident to a peak of the lattice envelop, an optical *Zitterbewegung* is obtained. If the optical beam is incident to a node of the envelop, a field localization effect takes place. The localized beam oscillates with a much larger spatial period than that of the optical *Zitterbewegung*. Variation of the oscillation period versus the split in periods is discussed. The position-dependent beam dynamics are explained by the excitation of proper bandedge eigenmodes of the Moiré lattice, and can be engineered via tuning the periods of the two superimposed Bragg lattices.

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## 1. Introduction

Governed by Maxwell's equations under the paraxial approximation, beam dynamics of electromagnetic waves in weakly modulated optical media have attracted much attention in recent years. The enthusiasm comes from the similarities between wave optics and quantum mechanics, which enables people to simulate many quantum effects of condensed matter and atomic systems in artificial optical nanostructures via the so-called quantum-optical analogues, see [1, 2] and references therein. Nowadays, with the introduction of concepts such as non-Hermitian Hamiltonian and parity-time symmetry, the study of quantum-optical analogues has been extended to optical systems with properly introduced loss and/or gain [3–8].

Most of the quantum-optical analogues investigated so far, e.g. Bloch oscillations, Zener tunnelling, and *Zitterbewegung* (ZB) [4, 9–15], are related to solid state physics [2]. To mimic the periodic potential experienced by electrons in a crystal, a periodic modulation on the refractive index of an optical material is utilized to realize a pre-defined quantum-optical analogue. However, nowadays we have also seen the importance of aperiodic structures such as Moiré gratings [16–20]. A Moiré grating is made of two superimposed Bragg gratings with different periods. Being totally different from an ordinary Bragg grating, the number of coupled waves propagating in a Moiré grating is infinite due to the cascading scattering by the two Bragg gratings [18]. Two stopbands are obtained near the resonances of the two Bragg gratings. They repel each other and result in the formation of a narrow slow-mode passband in between, making it conceptually analogous to the electromagnetically induced transparency [18, 21]. The slow-light effect of Moiré gratings also applies to surface plasmons on metallic surfaces, enabling the realization of a plasmonic cavity that localizes the propagating surface plasmon at its nodes and that can be utilized for lasing purpose [19, 20].

A Moiré grating [16–20], strictly speaking, is an one-dimensional structure because the propagating direction of field is parallel to that of periodic modulation. The concept can be extended to two-dimensional scenario in realizing a Moiré lattice as shown in Fig. 1(a), which is formed with two superimposed Bragg lattices. Optical field propagating inside it is mainly along the  $z$  direction, perpendicular to the  $x$  direction of periodicity. On one hand, beam dynamics in optical Bragg lattices with a single period have been intensively studied [2, 3, 5–7]. On the other hand, Moiré gratings have unique features such as slow light and field localizations [16–20]. It is then of great interest to see how a Moiré lattice modifies the propagating features of optical fields. Such a topic has not been seen in any literature, to the best of our knowledge.

In this article we investigate the beam dynamics in Moiré lattices. We show that if the oblique incident optical beam satisfies a quasi-Bragg condition, two kinds of interesting field trembling and oscillation effects can be observed depending on the position of incidence. At the nodes associated with a  $\pi$  phase shift between the two Bragg lattices, a strong field localization and oscillation is obtained. At the peaks associated with a zero phase shift, an optical ZB effect is observed. We show that these effects are associated with the excitation of different bandedge modes of the Moiré lattice. The field localization and oscillation are the main subject of this

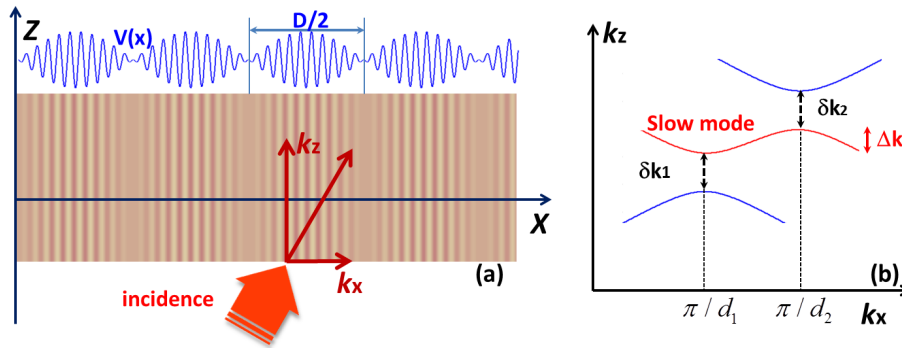


Fig. 1. (a) A schematic of the Moiré lattice under investigation. The media is infinite in the  $y - z$  plane, and in the  $x$  direction a weakly modulated optical potential  $V(x) = A[\cos(2\pi x/d_1) + \cos(2\pi x/d_2)]$  is introduced. Envelop of the Moiré lattice is characterized by a period of  $D$ . Parallel wavevector  $k_x$  of incidence is about the quasi-Bragg condition of  $k_x = \pi/\bar{d}$  in order to excite the bandedge modes. (b) A schematic of the band structure [18]. Two bandgaps with widths  $\delta k_1$  and  $\delta k_2$  are open at  $k_x = \pi/d_1$  and  $\pi/d_2$ , respectively. Between the two bandgaps a slow-mode branch with a width  $\Delta k$  is obtained.

investigation, which, we argue, is associated with the excitation of optical eigenmodes on the slow-mode branch. Its oscillation characteristics can be engineered by tuning the periods of the two superimposed Bragg lattices. Our investigation shows that band-structure engineering with the introduction of multiple periods can realize unique physical mechanism for various applications.

## 2. Theory and simulation

Figure 1 shows a schematic of the Moiré lattice under investigation. The medium is infinite in the  $y - z$  plane, and along the  $x$  direction a weak modulation  $V(x)$  on the optical potential is introduced,

$$V(x) = A\left[\cos\left(\frac{2\pi}{d_1}x\right) + \cos\left(\frac{2\pi}{d_2}x\right)\right], \quad (1)$$

where  $A$  is the amplitude of the modulation,  $d_1$  and  $d_2$  are periods of the two superimposed Bragg lattices ( $d_1 > d_2$ ). Assume  $d_1$  and  $d_2$  are close to each other

$$d_1 = d_0(1 + \delta) \text{ and } d_2 = d_0(1 - \delta), \quad (2)$$

where  $\delta \ll 1$ ,  $V(x)$  can be expressed to

$$V(x) = 2A \cos\left(\frac{2\pi}{D}x\right) \cos\left(\frac{2\pi}{\bar{d}}x\right). \quad (3)$$

Such a Moiré lattice is characterized by a high-frequency Bragg-like lattice with a period  $\bar{d}$  of

$$\bar{d} = 2 \frac{d_1 d_2}{d_1 + d_2} = d_0(1 - \delta^2) \quad (4)$$

enveloped by a low-frequency superimposed oscillation with a longer period  $D$  of

$$D = 2 \frac{d_1 d_2}{d_1 - d_2} = \frac{\bar{d}}{\delta}, \quad (5)$$

as can be seen from Fig. 1(a).

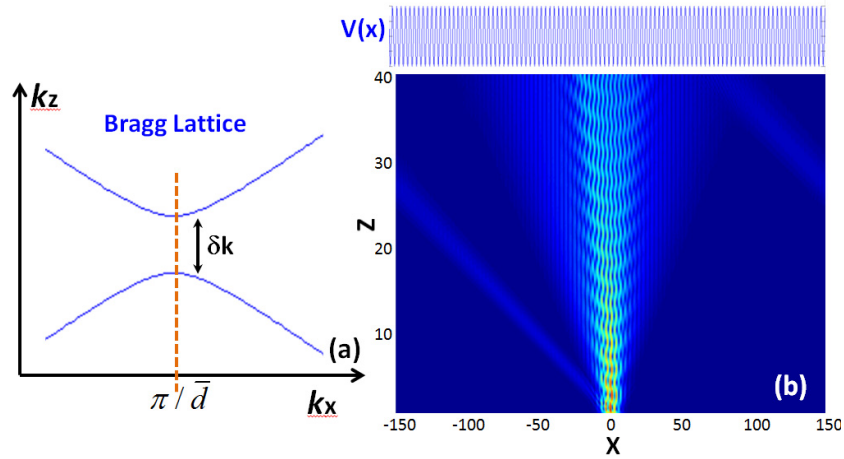


Fig. 2. (a) Schematic of the band structure of a Bragg lattice with  $V(x) = 2A \cos(2\pi x/d_0)$  near the zone edge. (b) When a beam is incident with  $k_x = \pi/d_0$ , eigenmodes around the anti-crossing are excited and produce ZB effect.

Assume the beam propagates mainly along the  $z$  direction, and the modulation  $V(x)$  is much weaker than the background [2–5, 15], the propagation dynamics of an optical field  $u$  is governed by a dimensionless equation of

$$i \frac{\partial u}{\partial Z} + \frac{\partial^2 u}{\partial X^2} + V(x)u = 0, \quad (6)$$

where the unitless parameters  $X$  and  $Z$  are defined by  $X = x/x_0$  and  $Z = z/x_0$ , with  $x_0 = \lambda/(4\pi n_0)$  the normalization length [3]. With Eq. (6) we program an algorithm code in MATLAB to simulate the propagation of optical beams in the Moiré lattice. With  $d_0 = 3x_0$  and  $\delta = 0.03$ ,  $\bar{d}$  is very close to  $d_0$ , and period  $D$  of the envelop  $V(x)$  is around  $100x_0$ . Coefficient  $A$  is assumed to be 1.

First, in order to provide a comparative analysis we simulate the situation of  $\delta = 0$ . Now a standard Bragg lattice of  $V(x) = 2A \cos(2\pi x/d_0)$  is formed. Figure 2 shows the distribution of field amplitude when an optical beam is oblique incident to the medium. Half width  $w_0$  of the incident beam is  $6x_0$ , and the parallel wavevector  $k_x$  equals  $\pi/d_0$ . Albeit the optical beam is oblique incidence, optical field inside the medium propagates mainly along the  $z$  direction because the Bragg condition of  $k_x = \pi/d_0$  renders an effective excitation of bandedge eigenmodes. Interference among the excited bandedge eigenmodes, the energy flux of which points mainly to the  $z$  direction, produces an optical ZB effect [14, 15, 22] characterized by a spatial trembling of field intensity, see Fig. 2(b).

Then, let us consider a Moiré lattice with  $\delta = 0.03$ . Figure 3 shows the results when the incident optical beam is relatively broad and has a half width  $w_0$  of  $100x_0$ . Both normal and oblique incidences are simulated. From Fig. 3(a) we can see that at normal incidence the optical field prefers to concentrate inside the waveguiding channels at peaks of  $V(x)$ , where the refractive index has a high value. This phenomenon is expected because bulk electromagnetic wave prefers optical denser medium. However, totally different features can be obtained when the beam is oblique incident to the Moiré lattice, especially when the parallel wavevector  $k_x$  satisfies the quasi-Bragg condition of

$$k_x = \frac{\pi}{\bar{d}} = \frac{\pi}{d_0(1 - \delta^2)}. \quad (7)$$

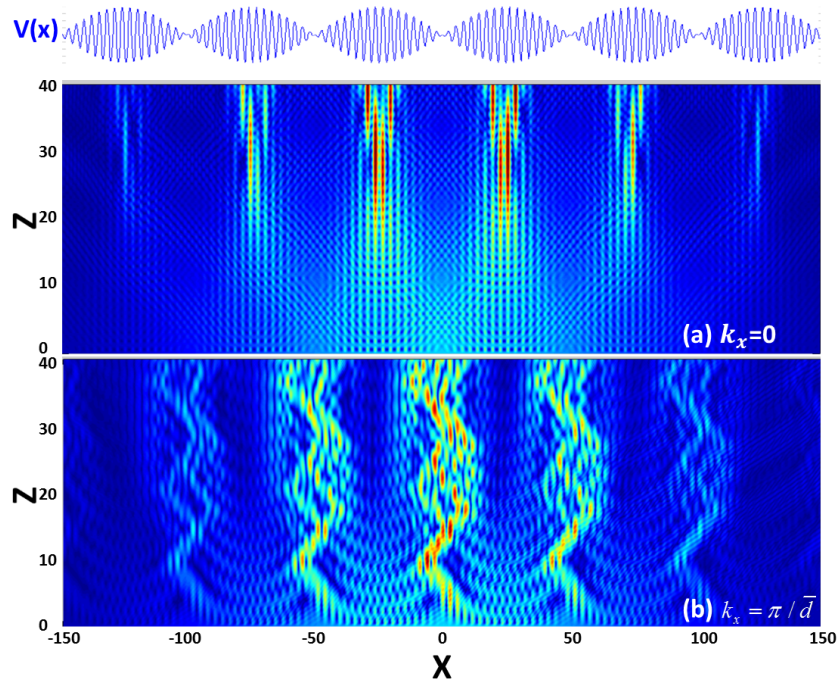


Fig. 3. Simulation results in a Moiré lattice of  $V(x) = A[\cos(2\pi x/d_1) + \cos(2\pi x/d_2)]$ . Half width  $w_0$  of the incident optical beams equal  $100x_0$ . (a) normal incidence with  $k_x = 0$ , and (b) oblique incidence with  $k_x = \pi/\bar{d}$ . Referring to  $V(x)$  plotted above we can see in (a) an obvious field enhancement is obtained around peaks of the Moiré lattice, while in (b) the field is enhanced at the nodes.

Here we term it a quasi-Bragg condition because  $\pi/\bar{d}$  is sandwiched between the Bragg conditions of  $k_x = \pi/d_1$  and  $k_x = \pi/d_2$  for the two constituent lattices. From Fig. 3(b) we can see the beam trembles in the Moiré lattice in a complex way. In each waveguiding channel characterized by the averaged period of  $\bar{d}$ , field trembles and periodically switches from one channel to an adjacent one [14, 15, 22]. Besides this ZB effect, by referring to  $V(x)$  plotted above we can observe an obvious field localization effect at nodes of the Moiré envelop  $|\cos(2\pi x/D)|$ . Oscillation of the localized field is much slower than ZB.

An optical beam with a broad width  $w_0$  could only excite eigenmodes within a narrow extension in wavevector  $k$  or angular frequency  $\omega$  space. To study the effect shown in Fig. 3(b) more clearly, we reduce the beam width  $w_0$  to  $6x_0$ . Figure 4(a) shows the scenario when the narrow optical beam is incident to a node of  $V(x)$ . We can see a much better field localization and oscillation behavior is obtained. The field package does not diverge, and oscillates inside the node. Period of the oscillation is around  $14x_0$  along  $z$  direction. Figure 4(b) shows the opposite scenario, where the position of incidence is at a peak of the envelop. In sharp contrast with Fig. 4(a), the beam diverges into a serial of waveguiding channels. Optical ZB can be observed in each waveguiding channel, and the trembling period is around  $3x_0$ .

The field localization and oscillation effect demonstrated in Fig. 4(a) is a general feature of Moiré lattices. We check the cases with different  $w_0$  values, e.g.  $w_0 = 2x_0$  and  $20x_0$ , and find that it can be still observed. However, the oscillation period is sensitive to the splitting parameter  $\delta$  of periods  $d_1$  and  $d_2$ . Figure 5 shows the variation of oscillation period versus  $\delta$ . We can see with decreased  $\delta$ , i.e. smaller different between periods  $d_1$  and  $d_2$ , the oscillation becomes slower and the period increases.

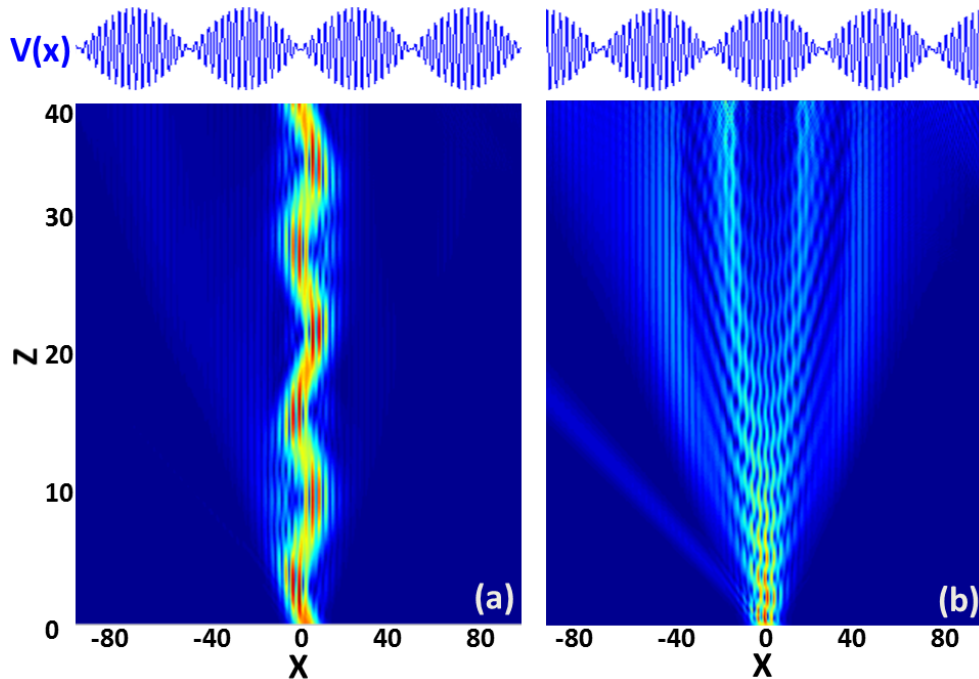


Fig. 4. Distributions of field intensity in the Moiré lattice for a narrow optical beam of  $w_0 = 6x_0$ , when incident to (a) a node, and (b) a peak of the envelop  $V(x)$ , respectively. The quasi-Bragg condition of  $k_x = \pi/\bar{d}$  is satisfied.

The oscillation effect shown in Fig. 4(a) is similar to Bloch and Bloch-Zener oscillations [4, 9–13]. However, Bloch oscillation is associated with the periodic motion of electrons when accelerated inside a crystal [4, 9–13]. To mimic the external field exerted over electrons via quantum-optical analogues, methods in achieving a suitable change of the waveguide characteristics are required, e.g. by using curved waveguide array [4, 12] or via temperature tuning [9]. Here the Moiré lattice is uniform in the  $y - z$  plane, and the beam is localized inside the nodes. All these features imply that the underlying physical mechanism is different from these of Bloch and Bloch-Zener oscillations [4, 9–13].

The beam dynamics present above should be closely related to the unique band structure of the Moiré lattice around the quasi-Bragg condition [16–19]. Figure 1 provides a schematic diagram of the band structure. A Moiré lattice is formed by superimposing two Bragg lattices with different periods of  $d_1$  and  $d_2$ . For a single Bragg lattice in the form of  $V(x) = A \cos(2\pi x/d)$ , the band structure at the bandedge  $k_x = \pi/d$  is characterized by a gap of  $\delta k$ , see Fig. 2(a). For a Moiré lattice shown in Fig. 1, in sharp contrast with that of a Bragg lattice, a narrow slow-mode branch is created between the two Bragg gaps at  $k_x = \pi/d_1$  and  $k_x = \pi/d_2$ . Since our simulation is set in the way that the quasi-Bragg condition of  $k_x = \pi/\bar{d}$  is satisfied, the incidence of a narrow beam, with a broad extension in  $k$  space, could efficiently excite bandedge modes covering the two Bragg gaps. In former literatures it has been shown that the excitation of slow mode can render a localization of surface plasmon in the nodes of a Moiré metallic surface [19, 20]. Since the unique beam dynamics shown in Fig. 4(a) is also characterized by a field localization inside a node, we expect that it should also be associated with the excitation of eigenmodes on the slow-mode branch of the Moiré lattice.

To verify above consideration, we propose to use Fourier expansion method to analyze the

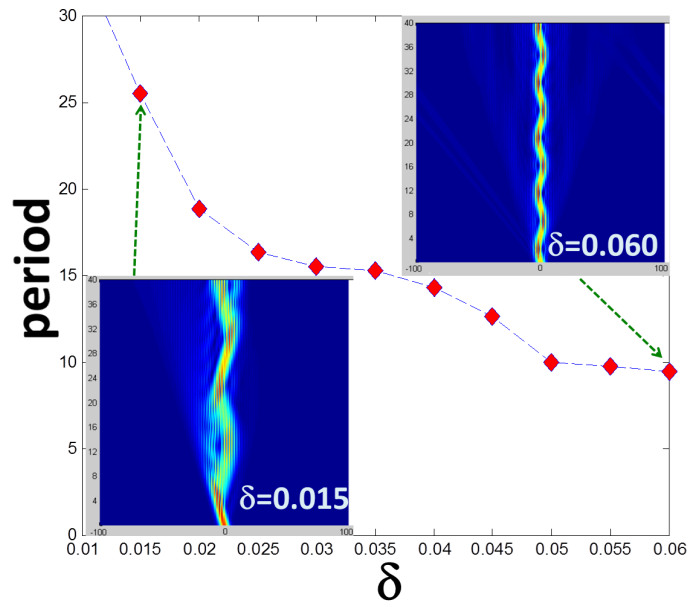


Fig. 5. Variation of oscillation period versus split  $\delta$  when incident to a node. Insets are the field distributions of  $\delta = 0.01$  and  $0.06$ .

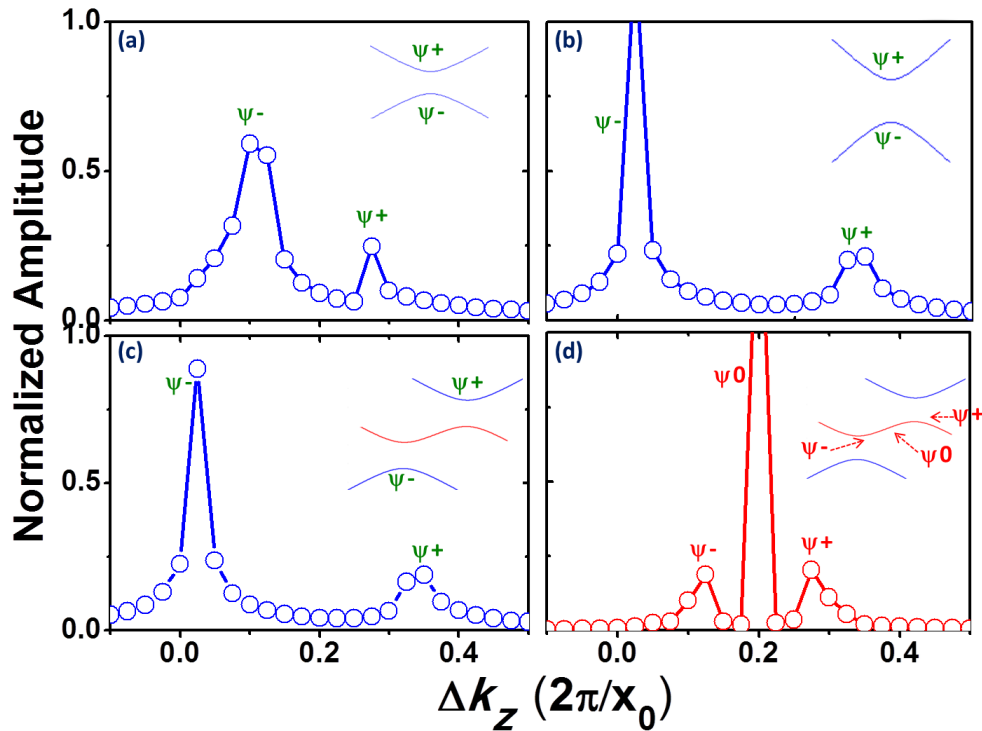


Fig. 6. Fast Fourier transform spectra on the complex  $u$  along the  $x = 0$  line in Bragg lattices (a)  $V(x) = A \cos(2\pi x/\bar{d})$ , (b)  $2A \cos(2\pi x/\bar{d})$ , and Moiré lattices  $A[\cos(2\pi x/d_1) + \cos(2\pi x/d_2)]$  when incident to (c) a peak and (d) a node, respectively. Insets are the schematic of band structures and distributions of the excited modes.

spectra of the complex field  $u$  along the line of  $x = 0$ . This method provides us information about the possible split harmonic components in the propagating field, from which we can briefly distinguish the bandedge structure and the distributions of excited eigenmodes. Fast Fourier transform results for the cases of Figs. 2 and 4 are shown in Fig. 6.

From Figs. 6(a) and 6(b) we can see for the Bragg lattices of  $V(x) = A \cos(2\pi x/\bar{d})$  and  $V(x) = 2A \cos(2\pi x/\bar{d})$ , two peaks are obtained in the Fourier spectra. The two peaks correspond to excited eigenmodes  $\Psi_{\pm}$  at the upper and lower branches of the bandedge, as schematically shown in the inset. The distance  $\delta k$  between the two peaks of  $V(x) = A \cos(2\pi x/\bar{d})$  [see Fig. 6(a)] is smaller than that of  $V(x) = 2A \cos(2\pi x/\bar{d})$  [see Fig. 6(b)], implying the existence of a narrower bandgap in the former case. ZB associated with a narrower bandgap at the zone edge would possess a larger trembling period of ZB [14, 15, 22], which is confirmed in our simulations.

Figures 6(c) and 6(d) show the Fourier spectra when incident to a peak and a node of the Moiré lattice  $V(x) = A[\cos(2\pi x/d_1) + \cos(2\pi x/d_2)]$ . When the beam is incident to a peak of  $V(x)$ , where the two superimposed Bragg lattices of  $\cos(2\pi x/d_1)$  and  $\cos(2\pi x/d_2)$  are in phase, a Fourier spectrum similar to that of  $V(x) = 2A \cos(2\pi x/\bar{d})$  is obtained, see Fig. 6(c). When the beam is incident to a node of  $V(x)$  that supports the field localization and oscillation effect shown in Fig. 4(a), we get a substantial different Fourier spectrum as shown in Fig. 6(d). Three sharp peaks are obtained, all of which are within the two split ones of Fig. 6(c). The central signal is much stronger than the adjacent two peaks.

Characteristics of Figs. 6(c) and 6(d) are in good agreement with a briefly analysis of the band structure from the coupled-mode theory developed in [18]. Albeit the number of coupled waves propagating in a Moiré lattice is infinite due to the cascading scattering by the two Bragg lattices [18], near the quasi-Bragg condition we can use only three Bloch components to express the eigenmodes  $\Psi$  by  $\Psi = [E_{inc}, E_{d1}, E_{d2}]^T$ , where  $E_{inc}$  represents the incident mode with a  $x$ -directional wavector of  $k_{inc}$ , while  $E_{d1}$  and  $E_{d2}$  are the modes with  $k_x = k_{inc} - 2\pi/d_1$  and  $k_x = k_{inc} - 2\pi/d_2$ , respectively, which are related to  $E_{inc}$  via the associated Bragg reflection. From the coupled-mode theory [18] we can find that in the middle slow-mode branch of the Moiré lattice,  $E_{d1}$  and  $E_{d2}$  possess a  $\pi$  phase difference, that

$$\text{phase}(E_{d1}) - \text{phase}(E_{d2}) = \pi. \quad (8)$$

Such a  $\pi$  phase difference between  $E_{d1}$  and  $E_{d2}$  agrees well with the phase difference between the two Bragg components of  $\cos(2\pi x/d_1)$  and  $\cos(2\pi x/d_2)$  at the nodes [19,20]. Consequently, when the optical beam is incident to a node, the  $\pi$  phase difference between  $\cos(2\pi x/d_1)$  and  $\cos(2\pi x/d_2)$  renders a  $\pi$  phase difference between the excited  $E_{d1}$  and  $E_{d2}$  components via the associated Bragg reflection from  $E_{inc}$ . Only eigenmodes on the slow-mode branch are excited. We can briefly figure out the correspondence between the Fourier signals and their relative positions in the slow-mode branch, as shown in the inset of Fig. 6(d). The middle peak  $\Psi_0$  is relatively high because it is dominated by the incident component  $E_{inc}$ .

As for the upper and lower branches, from the coupled-mode theory [18] we can find

$$\text{phase}(E_{d1}) - \text{phase}(E_{d2}) = 0. \quad (9)$$

With the same consideration presented above, when the optical beam is incident to peaks of  $V(x)$ , where  $\cos(2\pi x/d_1)$  and  $\cos(2\pi x/d_2)$  are in phase with each other, the excited  $E_{d1}$  and  $E_{d2}$  components also possess a zero phase difference, implying that only the eigenmodes on the upper and lower branches are excited. Now we arrive in a case very similar to that of a Bragg lattice  $V(x) = 2A \cos(2\pi x/\bar{d})$ , which explains the similarities between the Fourier spectra of Figs. 6(b) and 6(c). Above discussion also explains why the Fourier signals shown in Fig. 6(d) are between that of Fig. 6(c), because the slow-mode branch is sandwiched between the two Bragg bandgaps [18–21].

Above demonstrated field localization and oscillation effect in Moiré lattices shows that we can transfer some ideas in low-dimensional optical systems to high-dimensional ones for the realization of unique beam dynamics. It also proves that photonic band structure plays a key role in determining the propagating features of optical beams. Since the band structure can be engineered by changing parameters of the system, we can manipulate the beam dynamics in a versatile way. For the Moiré lattices investigated in this article, we can, for example, manipulating the field localization and oscillation by changing  $\delta$  that characterizes the split of periods  $d_1$  and  $d_2$ . When  $\delta$  decreases, the two Bragg gaps approach each other and make the slow-mode branch being flatter [18, 21]. It explains the results shown in Fig. 5, that with decreased  $\delta$ , the spatial oscillation period becomes larger.

The beam dynamics in Moiré lattices is different from the Bloch-Zener oscillations and ZB effect [4, 9–15]. It relies on the competition between two stop-bands associated with the superimposed Bragg lattices. Unlike ZB that can be observed in a single Bragg grating [15], the beam localization and oscillation effect in a Moiré lattice is observable only at the nodes. The  $\pi$  phase difference between the two Bragg components helps to realize a improved beam convergence and a longer propagating distance comparing with that of ZB [15]. It promises some potential applications, for example, the guiding and routing of optical signal in integrated optical circuits with possible all-optically manipulation. As for the experimental realization of the effects presented in this article, we can consider engineered arrays of waveguides [1, 2, 4], photo-refractive crystals [13], or the scanning phase mask technique present in [16]. The interference lithography method applied to a photosensitive polymeric surface reported in [19] also promises a possible observation of similar effects on surface waves.

### 3. Conclusion

In summary, in this article we study the propagation of optical beams in Moiré lattices. We demonstrate a position-dependent beam dynamics when the oblique incident beam satisfies the quasi-Bragg condition. When the optical beam is incident to a peak on the envelop of the optical Moiré lattice, an optical ZB can be observed. When the optical beam is incident to a node of the envelop, a field localization effect takes place. The position-dependent beam dynamics are explained by the excitation of proper bandedge optical eigenmodes of the Moiré lattice, especially these on the slow-mode branch. Variation of beam localization and oscillation versus split  $\delta$  in periods is investigated. Our study shows that beam dynamics in complex optical lattices can be engineered via tuning the periods of the composite Bragg lattices to realize many interesting effects.

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