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Compression of light pulses in photonic crystals

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Abstract. A study is made of the feasibility of controlling the phase and duration of short laser pulses with the aid of dispersive properties of photonic crystals. Pulse compression and phase modulation in one-dimensional structures with photonic band gaps are investigated analytically and numerically. It is shown that photonic crystals with a cubic optical nonlinearity can be used to reduce the duration of laser pulses, right down to several periods of an optical field, in structures with spatial dimensions typically less than a millimetre. The physical factors which limit the minimum duration of light pulses compressed in such photonic crystal structures are identified.

1. Introduction

Photonic crystals [1, 5] represent a new type of synthetic structurally organised media with a three-dimensional periodicity of the optical characteristics in which the crystallographic unit has dimensions of the order of one optical wavelength. Periodic modulation of the optical properties of photonic crystals results in special regimes of propagation of optical waves in certain intervals of wavelengths and wave vectors. In particular, interference of electromagnetic waves propagating along certain directions in such structures leads to the appearance of forbidden bands ('band gaps') of photon energies. If a structure with closed (complete) photonic band gaps (PBGs) is formed, waves with frequencies in a specific interval cannot propagate in such a structure irrespective of the directions of the wave and polarisation vectors. Such band gaps are analogues of the electronic energy gaps that appear in semiconductors.

Structures with PBGs have been under active investigation in recent years because of their wide range of applications, including control of spontaneous emission [5], development of vertical-cavity semiconductor lasers [6], and fabrication of Bragg reflectors and chirped mirrors [7], low-threshold optical switches and limiters [8], compact optical delay lines [9], nonlinear optical diodes [10], etc.

The feasibility of control of the group and phase velocities of optical pulses and of increasing the efficiency of nonlinear optical processes in PBG structures [11] are extremely important in the applications of photonic crystals as nonlinear optical frequency converters (second-harmonic generation in colloidal photonic crystals was reported [12]). Optical pulse compression and soliton formation are possible in fibre Bragg gratings as a result of the combined effect of the grating dispersion and self-phase modulation [13], as predicted theoretically earlier [14]. However, investigations of the propagation of light pulses in PBG structures have been limited mainly to the picosecond range of durations, so that it is not clear what are the ultimate parameters of ultrashort pulses that can be generated with the aid of photonic crystal structures.

We shall consider compression of short light pulses in linear and nonlinear photonic crystals. We shall employ approximate analytic expressions and numerical calculations for one-dimensional PBG structures to demonstrate that photonic crystals can be used to control the phase of short light pulses and that such pulses can be compressed over distances less than 1 mm.

2. Second-order approximation of the dispersion theory for slowly varying envelopes

The influence of dispersion on the propagation of short laser pulses in periodic structures which have band gaps in the optical range can be illustrated by considering the example of one-dimensional photonic crystals. A crystal of this type consists of a periodic alternation of layers with different refractive indices. We shall consider a one-dimensional photonic crystal consisting of alternate dielectric layers of thicknesses a and b, and with the refractive indices n_a and n_b, respectively. The dispersion relationship for such a crystal structure can be written in the form [15]

$$\cos(kd) = \cos\Big(\frac{\omega}{c}n_a a\Big)\cos\Big(\frac{\omega}{c}n_b b\Big) - \frac{n_a^2+n_b^2}{2n_a n_b}\sin\Big(\frac{\omega}{c}n_a a\Big) \times \sin\Big(\frac{\omega}{c}n_b b\Big), \quad (1)$$

where ω is the optical wave frequency; k is the wave number; c is the velocity of light; $d = a + b$ is the period of the PBG structure of this crystal.

We shall use the second-order approximation of the dispersion theory to study the influence of dispersion in a PBG structure on the duration of a laser pulse with a slowly varying envelope when this pulse propagates in a photonic crystal. We shall consider whether such a crystal can be used to control the phase and to compress phase-modulated laser pulses.

We shall assume that a phase-modulated light pulse has the Gaussian profile:

$$A_0(t) = \rho_0 \exp\big[-1/2(\tau_0^{-2} + i\alpha_0)t^2\big], \quad (2)$$

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where τ_0 is the characteristic pulse duration; α_0 is the rate of change of the carrier frequency of the pulse. In the approximation of slowly varying envelopes, when the second-order effects are included in the dispersion theory, the duration of a pulse with a profile described by the above expression and propagating in the investigated medium along the x axis can be described by [16]

$$\tau_p(x) = \tau_0 \left[\left(1 - \alpha_0 k_2 x\right)^2 + \left(\frac{k_2 x}{\tau_0^2}\right)^2 \right]^{1/2}, \qquad (3)$$

where $k_2 = \partial^2 k / \partial \omega^2$ is the group-velocity dispersion.

It follows from expression (3) that, in the ranges where

$$\alpha_0 k_2 > 0, \qquad (4)$$

the duration of a pulse with initial phase modulation first decreases, reaches its minimum value, and then increases. The minimum pulse duration

$$\tau_{\min} = \frac{\tau_0}{\left[1 + \left(\alpha_0 \tau_0^2\right)^2\right]^{1/2}} \qquad (5)$$

is then reached at a distance

$$L = \frac{\left(\alpha_0 \tau_0^2\right)^2}{\left[1 + \left(\alpha_0 \tau_0^2\right)^2\right] \alpha_0 k_2}. \qquad (6)$$

Both positive and negative dispersion of the group velocity is possible in photonic crystals. We shall illustrate the feasibility of efficient pulse compression in photonic crystals (modern technologies make it possible to fabricate photonic crystals with band gaps in the optical range [12, 17, 18]) by considering a one-dimensional PBG structure which obeys the dispersion relationship (1). The dependence of the parameter k_2 on the reciprocal wavelength λ^{-1} is shown in Fig. 1A for such a crystal with $d = 460$ nm, $a/b = 0.3$, $n_a = 1.0$, $n_b = 1.5$. In this case the wavelength $\lambda = 800$ nm (representing radiation of a titanium-doped sapphire laser) lies in the normal dispersion range of the group velocities characterised by $k_2 = 0.633$ fs^2 μm^{-1}. It follows from expressions (1)–(6) that such a PBG structure can reduce the duration of a phase-modulated pulse from its initial value $\tau_0 = 10T_0$ ($T_0 = 2\pi/\omega$ is the period of oscillations of the optical field and $\alpha_0 = 2/\tau_0^2$), to $8.27T_0$, $6.67T_0$, $5.38T_0$, $4.6T_0$ over distances 100, 200, 300, and 400 μm, respectively; the minimum duration, equal to $4.5T_0$, is reached at a distance of the order of 450 μm.

Anomalous dispersion of the group velocity of the $\lambda = 800$ nm radiation is possible in a one-dimensional PBG structure with the parameters $d = 460$ nm, $a/b = 0.3$, $n_a = 1.5$, $n_b = 1.0$ (Fig. 1B). In this case we have $k_2 = -1.122$ fs^2 μm^{-1}. It follows from expressions (1)–(6) that the duration of a pulse with $\tau_0 = 10T_0$ and $\alpha_0 = -2/\tau_0^2$ decreases to $6.3T_0$, $4.5T_0$, $6.3T_0$ over distances 127, 253, 380 μm, respectively, reaching a minimum duration of $4.5T_0$ at a distance of the order of 254 μm.

These estimates indicate that highly efficient compression of femtosecond pulses, typically over a spatial distance of less than 1 mm, is possible in photonic crystals. It follows that PBG structures are sufficiently promising as components of compact femtosecond laser systems. Such structures open up new avenues of miniaturisation of the current femtosecond solid-state systems (including titanium–sapphire lasers [19]).

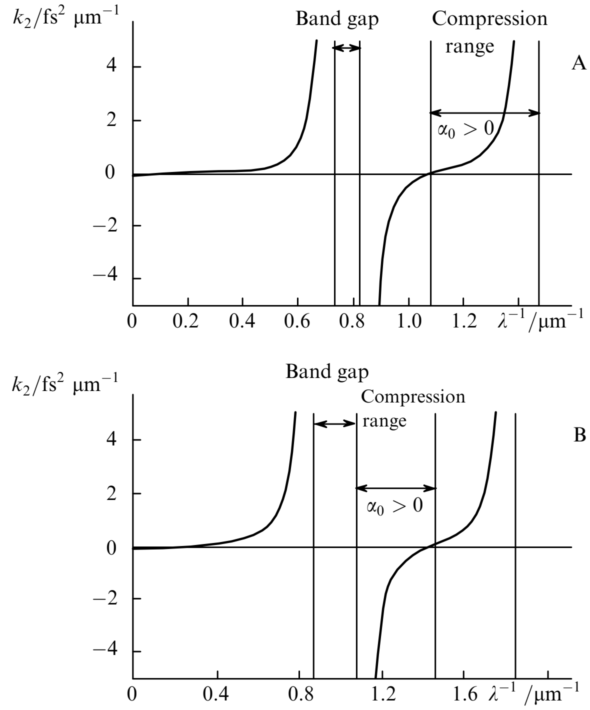

Figure 1. Dependences of the group-velocity dispersion k_2 on the reciprocal wavelength λ^{-1} for a one-dimensional photonic band-gap structure with $d = 460$ nm, $a/b = 0.3$, $n_a = 1.0$, (A) and 1.5 (B), $n_b = 1.5$ (A) and 1.0 (B). The compression ranges are identified for $\lambda = 800$ nm pulses.

3. Numerical analysis

Estimates of typical parameters of the compression of light pulses in a photonic crystal given in the preceding section were obtained in the second order of the dispersion theory for pulses with slowly varying envelopes. This approximation becomes invalid for light pulse durations of the order of several optical field periods. Nevertheless, the high-order dispersion may distort considerably the profiles of short laser pulses propagating in a photonic crystal.

These effects were taken into account by a procedure we developed for numerical modelling of the propagation of short light pulses in one-dimensional PBG structures. In accordance with our algorithm, a light pulse entering a photonic crystal is regarded as a wave packet. The corresponding Fourier integral obtained in the frequency representation is calculated by the fast Fourier transform method. Derivation of the direct fast transform for each Fourier component is followed by calculation of the envelopes of the electromagnetic fields transmitted by the photonic crystal. The envelope E_j of a given field in the jth layer (assumed to be uniform) is found in the form of a superposition of the reflected and refracted waves. For each jth layer with the permittivity $\varepsilon_j(x)$ the wave-vector components along the x axis, which is selected along the pulse propagation direction, can be written in the form

$$k_{jx}(x) = \varepsilon_j(x) \frac{\omega^2}{c^2}.$$

The electric field in the jth layer is represented by a superposition of the incident and reflected waves:

$$E_j = A_j \exp\left[i\left(k_{jx}x - \omega t\right)\right] + B_j \exp\left[i\left(-k_{jx}x - \omega t\right)\right],$$

where A_j and B_j are the envelopes of, respectively, the transmitted and reflected waves in the jth layer.

As in Section 2, we considered a one-dimensional PBG structure consisting of a periodic alternation of air and dielectric layers of thicknesses a and b, and with the refractive indices 1 and n, respectively. Introducing the notation $k_0 = \omega/c$, we can write down the relevant components of the wave vectors in the air and dielectric layers as $k_{\rm a} = k_0$, $k_{\rm d} = nk_0$. Application of the boundary conditions, which require continuity of the tangential components of the electric field and of the normal components of the electric induction, then yields the following system of equations for the envelopes of the fields in a one-dimensional PBG structure consisting of m air – dielectric layer pairs:

$$\begin{aligned} &E_{\rm r} - A_1 - B_1 = -E_0, \\ &k_{\rm a}E_{\rm r} + k_{\rm d}A_1 - k_{\rm d}B_1 = k_{\rm a}E_0\,, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &A_{2m}\exp({\rm i}k_{2mx}b) + B_{2m}\exp(-{\rm i}k_{2mx}b) - E_{\rm t} = 0, \\ &-k_{\rm d}A_{2m}\exp({\rm i}k_{2mx}b) + k_{\rm d}B_{2m}\exp(-{\rm i}k_{2mx}b) + k_{\rm a}E_{\rm t} = 0. \end{aligned} \tag{7}$$

Here, E_0 is the envelope of the incident field; $E_{\rm r}$ is the envelope of the field reflected from the crystal as a whole; $E_{\rm t}$ is the envelope of the field transmitted by the crystal.

A matrix $\hat{M}$ corresponding to the system of equations (7) is

$$\hat{M} =$$

$$\begin{pmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 \\ k_{\rm a} & k_{\rm d} & -k_{\rm d} & 0 & 0 & 0 & 0 & . & . & 0 & 0 & 0 \\ 0 & {\rm e}^{{\rm i}k_{\rm d}b} & {\rm e}^{-{\rm i}k_{\rm d}b} & -1 & -1 & 0 & 0 & . & . & 0 & 0 & 0 \\ 0 & -k_{\rm d}{\rm e}^{{\rm i}k_{\rm d}b} & k_{\rm d}{\rm e}^{-{\rm i}k_{\rm d}b} & k_{\rm a} & -k_{\rm a} & 0 & 0 & . & . & 0 & 0 & 0 \\ 0 & 0 & 0 & {\rm e}^{{\rm i}k_{\rm a}a} & {\rm e}^{-{\rm i}k_{\rm a}a} & -1 & -1 & . & . & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_{\rm a}{\rm e}^{{\rm i}k_{\rm a}a} & k_{\rm a}{\rm e}^{-{\rm i}k_{\rm a}a} & k_{\rm d} & -k_{\rm d} & . & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & {\rm e}^{{\rm i}k_{\rm d}b} & {\rm e}^{-{\rm i}k_{\rm d}b} & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & -k_{\rm d}{\rm e}^{{\rm i}k_{\rm d}b} & k_{\rm d}{\rm e}^{-{\rm i}k_{\rm d}b} & k_{\rm a} \end{pmatrix} \tag{8}$$

A determinant of order 4m, corresponding to this ribbon matrix with four codiagonals, can be calculated from the recurrence formula

$$D_{2m}^{+} = (k_{\rm d} + k_{\rm a})D_{2m-1}^{+}{\rm e}^{{\rm i}k_{\rm d}b} + (k_{\rm d} - k_{\rm a})D_{2m-1}^{-}{\rm e}^{-{\rm i}k_{\rm d}b}. \tag{9}$$

Here, D_{2m-1}^{+} is a determinant obtained by the $k_{\rm a} \leftrightarrow k_{\rm d}$ substitution from the determinant of the $(2m-1)$th order, which corresponds to a structure consisting of $2m-1$ layers. A determinant D_{2m-1}^{-} is obtained from D_{2m-1}^{+} if $k_{\rm d}$ is replaced with $-k_{\rm d}$.

We shall demonstrate the validity of formula (9) by the mathematical induction method. For a two-layer structure, consisting of one air and one dielectric layer, we have $D_0^{+} = k_{\rm a} + k_{\rm d}$. The determinant of an air – dielectric – air structure is

$$D_1^{+} = (k_{\rm d} + k_{\rm a})(k_{\rm a} + k_{\rm d}){\rm e}^{{\rm i}k_{\rm d}b} + (k_{\rm d} - k_{\rm a})(k_{\rm a} - k_{\rm d}){\rm e}^{{\rm i}k_{\rm d}b}.$$

The validity of formula (9) can easily be checked also for D_2^{+}, D_3^{+}, etc. If we assume that this formula applies for a certain value of m, we can show that this is valid also for $m+1$. The envelope of the transmitted wave $E_{\rm t}$ can be calculated from the Cramer rule:

$$E_{\rm t} = \frac{A}{D_{2m}^{+}}\,, \tag{10}$$

where A is a determinant obtained from D_{2m}^{+} by replacing the last column with one composed of the right-hand sides of the system of equations (7). The mathematical induction method makes it possible to prove also that

$$A = -2^{2m}k_{\rm a}^{m}k_{\rm d}^{m}E_0\,. \tag{11}$$

It follows that the algorithm described above can be used to find the envelopes of the fields transmitted by the PBG structure and this can be done for each Fourier component. When this procedure is applied to reconstruction of a pulse transmitted by a crystal, the inverse fast Fourier transform is obtained in the time domain.

4. Compression of a pulse and control of its phase

Figs 2A and 2B give the results of numerical calculations carried out in accordance with the procedure described in the preceding section. This is done for PBG structures which are optimised (in the sense of selection of the operating range; see Section 6) for laser radiation pulses of the wavelengths 0.76 and 0.80 μm ($a/b = 0.3$, $d = 0.46$ μm, $n = 1.5$). It is evident from Fig. 2A that a pulse which propagates in the investigated PBG structure and is free of the initial chirp ($\alpha_0 = 0$) spreads out in time and experiences phase modulation. A suitable selection of the initial phase modulation can ensure efficient compression of this pulse by compensation of the chirp during propagation of the pulse through a photonic

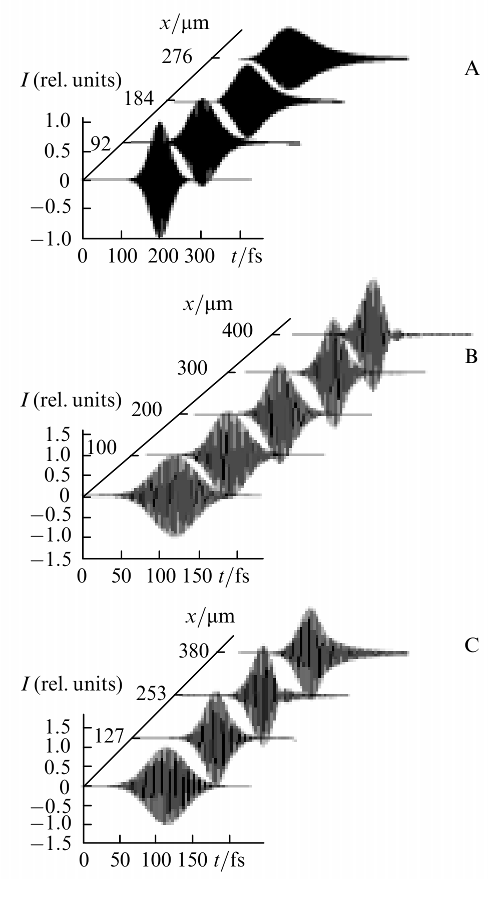

Figure 2. Phase control and compression of $\lambda = 760$ nm laser pulses propagating in a one-dimensional PBG structure with $a/b = 0.3$: (A) spreading of a laser pulse for $d = 490$ nm, $n_a = 1$, $n_b = 1.5$; (B, C) compression of phase-modulated pulses for $d = 460$ nm, $n_a = 1$, $n_b = 1.5$ (B) and $d = 460$ nm, $n_a = 1.5$, $n_b = 1$ (C).

crystal (Figs 2B and 2C). Variation of the parameters of the PBG structure (lattice constant d, fill factor a/d, and ratio of the refractive indices of the materials forming the photonic crystal) provides an opportunity for control of the dispersion of the crystal and, consequently, for control of the parameters (duration and phase) of short laser pulses.

It follows that photonic crystals can be used for controlled compression of short laser pulses over very short distances. The results presented in Fig. 2B demonstrate that the duration of a phase-modulated laser pulse of the wavelength $\lambda = 800$ nm, of initial duration $\tau_0 = 10T_0$, and characterised by $\alpha_0 = 2/\tau_0^2$, can be shortened in a photonic crystal whose parameters are $d = 460$ nm, $a/b = 0.3$, $n_a = 1.0$, $n_b = 1.5$ (with normal group-velocity dispersion; Fig. 1A) to $4.5T_0$ over a distance of 450 μm. If this distance is $x = 100$, 200, 300, 400 μm, the pulse duration becomes $8T_0$, $6.3T_0$, $4.8T_0$, respectively. In a photonic crystal with the parameters $d = 460$ nm, $a/b = 0.3$, $n_a = 1.5$, $n_b = 1.0$ (with anomalous group-velocity dispersion; Fig. 1B) the duration of a pulse with $\lambda = 800$ nm, $\tau_0 = 10T_0$, and $\alpha_0 = -2\tau_0^2$ decreases to $5.5T_0$, $5T_0$, $5.5T_0$ for $x = 127$, 253, 380 μm, respectively (Fig. 2C).

It follows that the results of the numerical calculations of the maximum compression length and of the minimum pulse duration, and also of changes in the pulse duration in the course of propagation in a PBG structure are in good agreement with the estimates obtained within the framework of the second-order dispersion theory for slowly varying envelopes (see Section 2). However, it is clear from Figs 2A – 2C that the approximation represented by expressions (1) – (6) is insufficiently accurate to describe evolution of a laser pulse when the pulse duration becomes of the order of several optical field periods, and also near the edges of the band gap where high-order dispersion effects distort considerably the profile of a light pulse. Such profile distortions, which appear beginning from distances of the order of 300 μm (Fig. 2B) and 250 μm (Fig. 2C), cannot be described in the approximation of slowly varying envelopes in terms of the spatial coordinates when only the first- and second-order dispersion is taken into account.

5. Pulse compression in nonlinear photonic band gap structures

In a photonic crystal considered above, the process of laser pulse compression reduces to compensation of the initial chirp. The minimum pulse duration which can be achieved in such a compressor in the absence of an optical nonlinearity is of the order of the duration of a transform-limited pulse (if this is permissible in the operating spectral range of the PBG structure; for details see Section 6). In the present section we shall show that true compression of light pulses, i.e. a reduction of their duration to less than the duration of a transform-limited input pulse, is possible in PBG structures with a cubic optical nonlinearity.

Chirp compensation in such structures makes it possible to compress not only phase-modulated laser pulses, but also pulses without initial phase modulation. A photonic crystal compressor of this type consists of alternate nonlinear-optical layers of thicknesses a and b, and with the permittivities

$$\varepsilon_a = \varepsilon_{a0} + \varepsilon_{\rm nl}|E|^2,$$
$$\varepsilon_b = \varepsilon_{b0} + \varepsilon_{\rm nl}|E|^2$$

respectively. Such a system combines the dispersion properties typical of PBG structures with a possibility of nonlinear-optical transformation of the spectrum of a laser pulse.

A pulse propagating in such a medium is broadened spectrally by self-phase modulation. A suitable selection of the sign of the product of the cubic susceptibility, responsible for self-phase modulation, and the group-velocity dispersion represented by k_2 makes the velocity of propagation of the leading edge of a pulse less than the velocity of propagation of the trailing edge, so that true compression of a light pulse is possible.

Compression of a pulse in a PBG structure with a cubic nonlinearity is attainable subject to the condition (see, for example, Ref. [16])

$$\varepsilon_{\rm nl}\frac{\partial^2 k}{\partial\omega^2} < 0. \qquad (12)$$

It follows that pulse compression can be achieved in the regions with anomalous dispersion of the group velocity when $\varepsilon_{\rm nl} > 0$ and in the regions of normal dispersion if $\varepsilon_{\rm nl} < 0$. Although analytic expressions for self-compression of light pulses, obtained in the approximation of the second order of the dispersion theory for pulses with slowly varying envelopes [16], can be used to determine the conditions for pulse compression and even (in certain situations) to estimate typical parameters of pulse compression, these expressions do not provide a satisfactory description of the influence of the

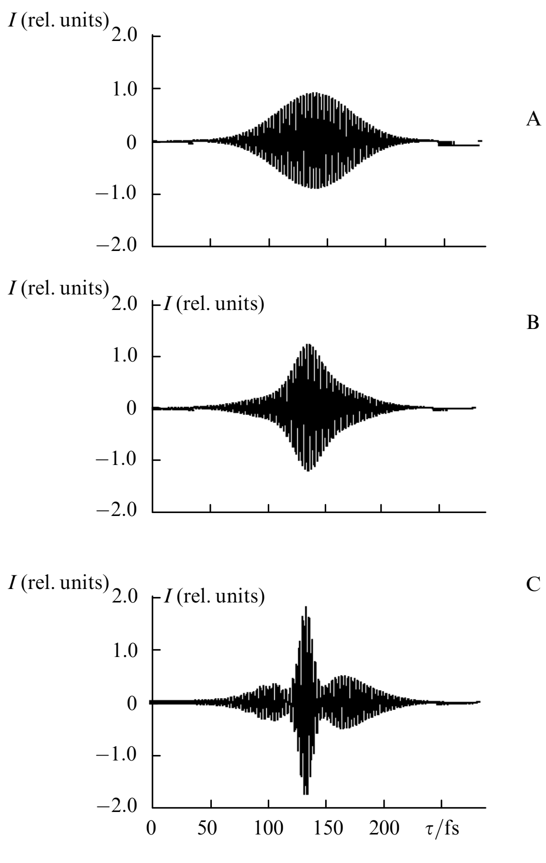

Figure 3. Propagation of a laser pulse with an initial duration $\tau_0 = 10T_0$ in a linear one-dimensional photonic band-gap structure with $a = b$, $\sqrt{\varepsilon_{a0}} = 1$, $\sqrt{\varepsilon_{b0}} = 1.5$, $d = 0.41$ μm, $\varepsilon_{\rm nl}|E|^2 = 0$ (A), 0.002 (B), and 0.004 (C). Structure length $l = 600d$ ($\tau = l/c - t$).

higher-order dispersion on the pulse profile and they become invalid for ultrashort laser pulses of duration which does not permit application of the approximation of slowly varying envelopes.

Fig. 3 gives the results of numerical modelling of the propagation of a short light pulse of the $\lambda = 800$ nm wavelength in a nonlinear-optical PBG structure with the following parameters: $a = b$, $\varepsilon_{a0} = 1$, $n_0 = \sqrt{\varepsilon_{b0}} = 1.5$, $d = 0.41$ µm. The procedure used in numerical calculations was based on the nonlinear finite-difference time-domain technique (see, for example, Ref. [20]). The results of our numerical modelling showed that such PBG structures can compress laser pulses efficiently. In particular, in the absence of nonlinearity the pulse duration $\tau_0 = 10T_0$ increases in a PBG structure to $11.5T_0$ (Fig. 3A) and it can be reduced to $5.5T_0$ and $2T_0$ (subject to a considerable distortion of the profile in the latter case) in PBG structures with $\varepsilon_{\rm nl}|E|^2 = 0.002$ (Fig. 3B) and 0.004 (Fig. 3C) over a distance $600d$. If $d = 520$ nm (as in the case of photonic crystal structures described in Ref. [18]), efficient compression of pulses to several periods of the optical field occurs over distances less than a millimetre.

Numerical calculations demonstrate clearly the influence of the higher-order dispersion which may distort considerably the profile of a light pulse, can give rise to a pedestal, and may even split a pulse into several spikes. In view of this it is necessary to optimise the parameters of a PBG compressor in order to achieve a compromise between the compression efficiency and distortion of the pulse profile.

6. Ultimate pulse duration

As shown in the preceding section, in contrast to linear photonic crystals, nonlinear PBG structures make it possible to generate pulses of duration less than that of transform-limited pulses at the entry to a structure. However, the minimum pulse duration attainable in linear and nonlinear PBG structures is limited by the width of the spectral range at the edge of the band gap, where the group velocity dispersion has the required sign, so that condition (4) is satisfied for a linear PBG structure or condition (12) for a nonlinear structure. Outside this range the spectrum of a laser pulse is either inside the band gap or in the range where the group-velocity dispersion results in spreading of the pulse. An increase in the width of this range (operating range) makes it possible, on the one hand, to generate shorter pulses with a given PBG structure and, on the other, it reduces the group-velocity dispersion and, consequently, increases the compression length.

It follows that a photonic crystal compressor should be optimised to ensure the minimum pulse duration at the output for a reasonable crystal length. Figs 4A and 4B give the dependences of the minimum pulse duration $\tau_{\min}$ and of the dispersion length $L_{\rm d}$ on the ratio a/b, obtained for various refractive indices $n_b = n$ (it is assumed that $n_a = 1$). It follows from these dependences that, as the ratio a/b increases to about 0.3, the minimum pulse duration increases slowly, whereas the dispersion length becomes considerably shorter because of an increase in the group-velocity dispersion. Some reduction in the minimum pulse duration by an increase in the ratio a/b is fully justified in this region, since this reduces considerably the length in which the maximum pulse compression is achieved. A higher a/b ratio increases further the minimum pulse duration, whereas the dispersion length depends much less on the ratio a/b. A reduction in the maximum-compression length can be achieved in this range only at the expense of a considerable increase in the minimum pulse duration.

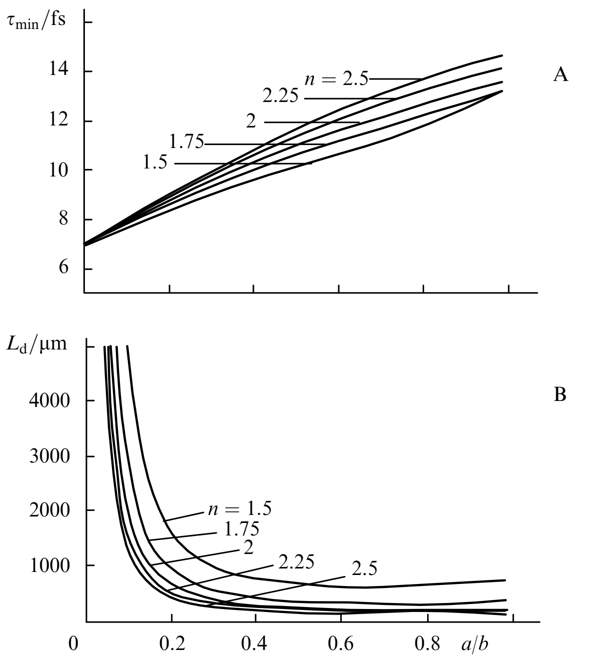

Figure 4. Dependences of the minimum pulse duration $\tau_{\min}$ (A) and of the dispersion length $L_{\rm d}$ (B) on the ratio a/b for various refractive indices n.

7. Conclusions

Our investigation shows that the dispersive properties of photonic crystals make it possible to control the phase and compensate the chirp of short light pulses. Estimates obtained for one-dimensional PBG structures and the results of numerical calculations carried out for linear and nonlinear photonic crystals with the lattice parameters and the ratio of the refractive indices close to those found experimentally show that PBG structures can be used to control the phase and compression of pulses over submillimetre distances. Combination of self-phase modulation and control of the dispersion of PBG structures composed of photonic crystals characterised by a cubic optical nonlinearity makes it possible to achieve true compression of ultrashort pulses to several optical field periods. The minimum duration which can be achieved by compression of light pulses in PBG structures is limited by the width of the spectral range at the band gap edge where the group velocity dispersion is of the required sign. This dispersion compensates the chirp in a linear PBG structure or causes self-compression of pulses in a nonlinear PBG structure.

The results of our investigations show that linear and nonlinear photonic crystals can be regarded as promising optical components which can help in modern femtosecond technologies, so that compact femtosecond laser systems can be constructed. In particular, preparation of photonic crystals with the required properties makes it possible to construct compact compressors of light pulses that should be useful in miniaturisation of advanced solid-state femtosecond laser systems.

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References

1. Yablonovitch E Phys Rev Lett. **58** 2059 (1987)
2. John S Phys. Rev. Lett. **58** 2486 (1987)
3. Yablonovitch E, Gmitter T J Phys. Rev. Lett. **63** 1950 (1989)
4. Yablonovitch E J. Opt. Soc. Am. B **10** 283 (1993)
5. Soukoulis C M (Ed.) Photonic Gaps and Localization (New York: Plenum Press, 1993)
6. Deppe D G, Lei C J. Appl. Phys. **70** 3443 (1991); Yamamoto Y, Machida S, Bjork G Phys. Rev. A **44** 657 (1991); Yokoyama H, Nishi K, Anan T, Yamada H, Brorson S D, Ippen E P Appl. Phys. Lett. **57** 2814 (1990)
7. Baba T, Hamano T, Koyama F, Iga K IEEE J. Quantum Electron. **27** 1347 (1991); Stingl A, Lenzner M, Spielmann C, Krausz F, Szipöcs R Opt. Lett. **20** 602 (1995); Xu L, Spielmann C, Krausz F, Szipöcs R Opt. Lett. **21** 1259 (1996); Kopf D, Prasad A, Zhang G, Moser M, Keller U Opt. Lett. **22** 621 (1997); Mayer E T, Möbius J, Euteneuer A, Rühle W W, Szipöcs R Opt. Lett. **22** 528 (1997)
8. Scalora M, Dowling J P, Bowden C M, Bloemer M J Phys. Rev. Lett. **73** 1368 (1994); Radic S, George N, Agrawal G P J. Opt. Soc. Am. B **12** 671 (1995)
9. Scalora M, Flynn R J. Reinhardt S B, Fork R L, Bloemer M J, Tocci M D, Bendickson J, Ledbetter H. Bowden C M, Dowling J P, Leavitt R P Phys. Rev. E **54** 2799 (1996)
10. Tocci M D, Bloemer M J, Scalora M, Dowling J P, Bowden C M Appl. Phys. Lett. **66** 2324 (1995)
11. Sakoda K, Ohtaka K Phys. Rev. B **54** 5742 (1996); Shimano R, Inouye S, Kuwata-Gonokami M, Nakamura T, Yamanishi M, Ogura I Jpn. J. Appl. Phys. **34** L817 (1995); Scalora M, Bloemer M J, Manka A S, Dowling J P, Bowden C M, Viswanathan R, Haus J W Phys. Rev. A **56** 3166 (1997); Hattori T, Tsurumachi N, Nakatsuka H J. Opt. Soc. Am. B **14** 348 (1997)
12. Trull J, Vilaseca R, Martorell J, Corbalan R Opt. Lett. **20** 1746 (1995); Martorell J, Corbalan R, Vilaseca R, Trull J, in Photonic Band Gap Materials (Ed. C M Soukoulis) (Amsterdam: Kluwer Academic, 1996) p. 529; Martorell J, Vilaseca R, Corbalan R Appl. Phys. Lett. **70** 702 (1997)
13. Eggleton B J, Slusher R E, de Sterke C M, Krug P A, Sipe J E Phys. Rev. Lett. **76** 1627 (1996)
14. Winful H G Appl. Phys. Lett. **46** 527 (1985)
15. Brekhovskikh L M Waves in Layered Media 2nd edition (New York: Academic Press, 1980); Yariv A, Yeh P Optical Waves in Crystals (New York: Wiley, 1983)
16. Akhmanov S A, Vysloukh V A, Chirkin A S Optics of Femtosecond Laser Pulses (New York: American Institute of Physics, 1992)
17. Cheng C C, Scherer A J. Vac. Sci. Technol. B **13** 2696 (1995); Cheng C C, Scherer A, Arbet-Engels V, Yablonovitch E J. Vac. Sci. Technol. B **14** 4110 (1996); Noda S, Yamamoto N, Sasaki A Jpn. J. Appl. Phys. 2 **35** L909 (1996); Knight J C, Birks T A, Russell P St J, Atkin D M Opt. Lett. **21** 1547 (1996); Lin H-B, Tonucci R J, Campillo A J Opt. Lett. **23** 94 (1998)
18. Bogomolov V N, Gaponenko S V, Kapitonov A M, Prokofiev A V Ponyavina A N, Silvanovich N I, Samoilovich S M Appl. Phys. A **63** 613 (1996); Vlasov Yu A, Astratov V N, Karimov O Z, Kaplyanskii A A, Bogomolov V N, Prokofiev A V Phys. Rev. B **55** R13357 (1997)
19. Kaertner F X, Matuschek N, Schibli T, Keller U, Haus H A, Heine C, Morf R, Scheuer V, Tilsch M, Tschudi T Opt. Lett., **22** 831 (1997); Sartania S, Cheng Z, Lenzner M, Tempea G, Spielmann Ch, Krausz F, Ferencz K Opt. Lett. **22** 1562 (1997)
20. Goorjian P M, Taflove A Opt. Lett. **17** 180 (1992); Tran P Opt. Lett. **21** 1138 (1996)