



## Short Communication

## Topological boundary states transport in synthetic four-dimensional acoustic system

Chudong Xu<sup>a,b</sup>, Taotao Zheng<sup>a</sup>, Hao Ge<sup>b</sup>, Wei Wang<sup>c</sup>, Ze-Guo Chen<sup>d,\*</sup>, Ming-Hui Lu<sup>b,e,\*</sup>, Yan-Feng Chen<sup>b</sup><sup>a</sup> College of Electronic Engineering (College of Artificial Intelligence), South China Agricultural University, Guangzhou 510642, China<sup>b</sup> College of Engineering and Applied Sciences and National Laboratory of Solid State Microstructures, Nanjing University, Nanjing 210093, China<sup>c</sup> Department of Physics, Hong Kong Baptist University, Hong Kong, China<sup>d</sup> School of Materials Science and Intelligent Engineering, Nanjing University, Suzhou 215163, China<sup>e</sup> Jiangsu Key Laboratory of Artificial Functional Materials, Nanjing 210093, China

## ARTICLE INFO

## Article history:

Received 27 May 2022

Received in revised form 25 July 2022

Accepted 29 August 2022

Available online 5 September 2022

© 2022 Science China Press. Published by Elsevier B.V. and Science China Press. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

Topological phase is an important development and unexplored degree of freedom of traditional band theory [1]. Generally, it manifests in physics through the boundary states revealed by the bulk-edge correspondence. Kaleidoscopic bulk topology implies various low-dimensional states at the boundary, giving extraordinary approaches to manipulate waves. Another structure that encodes the band topology is the system associated with space–time evolution, such as Floquet topological insulator [2,3]. Its quasistatic limit involves time-dependent dynamic adiabatic evolution [4,5]. However, dynamic evolution is challenging as it requires fast modulation to overcome the dissipation of state before it is pumped from one edge to another [6]. An alternative approach is to use an additional degree of freedom to replace the role of time, and an illuminating system is the weakly coupled waveguides, whose governing equation can be directly mapped to Schrödinger equation with time modulation replaced by wave propagation along the waveguides. Recently, such strategy has been largely explored to realize the pumping of topological boundary states in optics [7,8] and elastic wave [9,10]. However, the strategy for acoustics is subtly different as the coupling of the acoustic waveguides is not weak for the fundamental waveguide mode. Thus exploring the possibility to realize two-dimensional (2D) topological acoustic pumping in a continuous regime remains an open question.

Here we theoretically propose and experimentally verify a simple continuous design to realize higher-order topological state pumping in acoustics. The system comprises straight waveguides with coordinate-dependent coupling waveguides that contribute two synthetic dimensions, making our design a four-dimensional

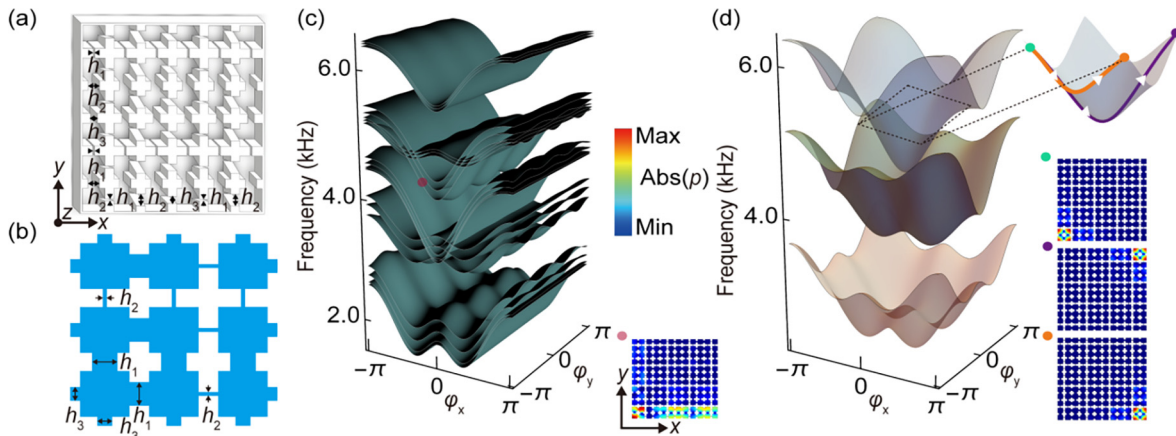
(4D) system. The topological pumping incorporates adiabatic modulation of continuously varied coupling waveguides, rendering as abstract trajectories in the synthetic dimensions. With the control over the phason in the synthetic space and the excitation by the corresponding sound source mode, we have realized the real space localized wave manipulating, such as robust directional edge-to-edge and corner-to-corner transport. Our synthetic system achieved through continuous modulations provides an effective way for 2D topological acoustic pumping for exploring the higher-order and high-dimensional topological phase to control wave propagation in three dimensions.

The designed sample is schematically illustrated in Fig. 1a and the corresponding unit-cell is shown in Fig. 1b. The adiabatic variation of phase parameters is realized by continuously modulating the coupling waveguides. To be specific, the modulating channel thickness  $h$  is a function of  $z$ , written as  $h_n^\mu = h_0 + \delta h \cos(2\pi bn + \varphi_\mu(z))$  ( $n = 1, 2, 3$ ),  $\mu = x$  or  $\mu = y$ . The modulation has a phase factor  $\varphi_x$ , which can be regarded as a pseudomomentum that constitutes a synthetic dimension in our system. When the phase parameters  $\varphi_x$  and  $\varphi_y$  linearly depend on  $z$ , the channel thickness follows cosine modulation profile. The system holds both 4D quantum Hall physics and higher-order topology [7,11], which manifests as edge states and corner states in 2D real space, respectively. More details about design method are provided in the [Supplementary materials](#). We consider a system containing  $12 \times 12$  cavities, the eigenfrequency spectra as functions of  $\varphi_x$  and  $\varphi_y$  of edge (corner) states are plotted in Fig. 1c (d). The simulated eigenfields are shown in the right panel to verify the existence of the corresponding edge states and corner states.

Since these topological states in the 2D real space systems are labelled with synthetic coordinates  $(\varphi_x, \varphi_y)$ , we can redistribute

\* Corresponding authors.

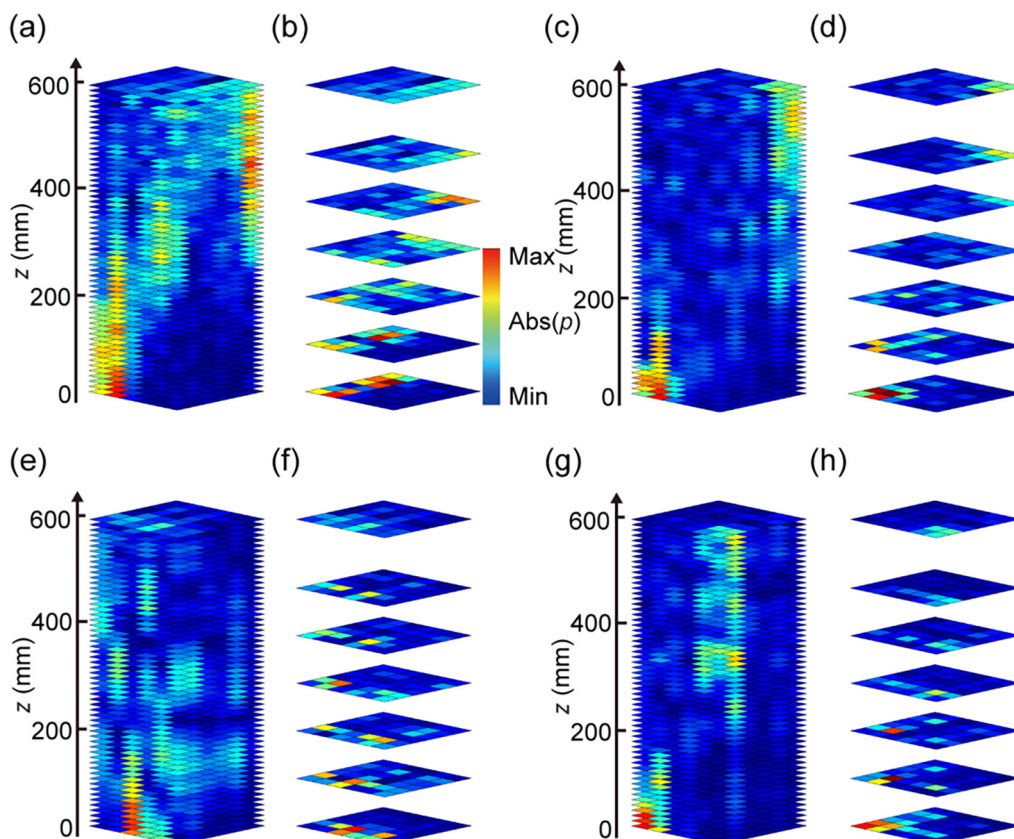
E-mail addresses: [zeguoc@foxmail.com](mailto:zeguoc@foxmail.com) (Z.-G. Chen), [luminghui@nju.edu.cn](mailto:luminghui@nju.edu.cn) (M.-H. Lu).



**Fig. 1.** (Color online) Schematic of the design sample and the eigenfrequency spectra. (a) Schematic of channel-modulated acoustic structure. (b) Unit-cell of 2D topological pumping system with  $3 \times 3$  sites. (c) The eigenfrequency spectra of edge modes plotted as functions of  $\varphi_x$  and  $\varphi_y$ . Right: The simulated edge states eigenfields of 2D finite acoustic array containing  $12 \times 12$  sites with the eigenfrequency  $f = 4002$  Hz. The brown circle points indicate the synthetic coordinates. (d) The eigenfrequency spectra of corner modes. Zoom-in shows eigenfrequency spectra of the highest frequency band partially extracted from Fig. 1c. The purple line and orange line describe two distinct phase orbits. Right: The simulated corner states eigenfields with the frequency  $f = 5839$  Hz. The green, purple and orange circle points indicate the synthetic coordinates.

these topological boundary states according to the designed phason paths. We consider two specific configurations. In configuration A, the variation of the synthetic coordinate follows a trajectory from  $(-0.5\pi, -0.5\pi)$  to  $(0.5\pi, 0.5\pi)$ , as shown by the purple line in Fig. 1d; in configuration B, it gradually turns from  $(-0.5\pi, -0.5\pi)$  to  $(0.5\pi, -0.5\pi)$  while keeping  $\varphi_y$  constant, as shown by the orange line in Fig. 1d.

We now investigate the topological state pumping in the above two designed samples, which are fabricated by the stereolithographic 3D printing technique. More details about the two samples and experimental method are provided in the [Supplementary materials](#). As we elaborated above, the system supports multiple topological edge states and corner states at each cut-plane with distinct frequency. In order to drive these states propagating along



**Fig. 2.** (Color online) Topological pumping of two designed samples. (a–h) Experimental observation of edge-to-edge and corner-to-corner pumping for the sample A (a–d) and the sample B (e–h), respectively. The sample A and the sample B are designed to evolve along the purple line and the orange line in Fig. 1d, respectively.

z direction, the propagation constant must be real, which requires the exciting frequency to be higher than the eigenfrequency of these states in 2D cut-plane. We set the working frequency to be 6000 Hz, which is sufficient to excite the edge states and corner states of our interest in this study. In Fig. 2a, we demonstrate experimentally the predicted edge-edge transport in our real three-dimensional (3D) acoustic sample A. We also plot the acoustic field at several cut-planes to better show the edge-to-edge transport in Fig. 2b. As we can see from Fig. 2a, b, the acoustic field indeed transports from one edge to another edge as it propagates along z direction. The sample is able to support corner-to-corner transport as well. As shown in Fig. 2c, d, the state evolves from the bottom-left corner at the input end to the upper-right corner of the output end, which reveals that the distribution of sound pressure field indeed renders the topological pumping process in the physical dimensions. So far our results evidently show potential to manipulate the acoustic state in 3D real structure.

By adjusting the given trajectory in phase space, the pumping processes are directional, which offers additional degree of freedom to control waves in 3D real space. As for sample B, since  $\varphi_y$  is unchanged, the edge state will not change its localization in y direction. The edge state contributed by separable one-dimensional model in y direction will keep at the same edge, which is verified by experiments as shown in Fig. 2e, f. Also, the corner states transport path swaps from the original bulk diagonal to the plane diagonal at the same side of our sample, as shown in Fig. 2g, h. Note that all the above experimental results are validated by the numerical simulations, whose details are provided in the [Supplementary materials](#).

In conclusion, we have experimentally verified a method to drive acoustic wave in 3D structures along a predefined path by utilizing 4D topology. The synthetic 4D system is contributed by introducing two synthetic dimensions through the phason encoded in continuously varied waveguides, which are feasible to fabricate. Compared to the 4D topology-induced boundary states in synthetic dimension labelled samples [12], we study the boundary states evolution in an integrated sample. The 4D topology is revealed by boundary states transferring. The implementation of edge-to-edge and corner-to-corner transport rely on adiabatic modulation of states along a given orbit in the phason space. Future work may explore the non-adiabatic evolution to speed up the pumping rate and shorten the modulation length with fast-driven pumping cycles [13]. It's worth noting that the topological boundary states to be pumped are not necessarily confined to the boundary of the real 3D structure. The topological states may manifest as waveguided states in the 3D bulk protected by employing only one synthetic dimension but three spatial dimensions to construct 4D topology [14,15]. We believe that our work provides a reference to realize a controllable, programmable and robust acoustic wave transport by employing high-dimensional topological physics.

### Conflict of interest

The authors declare that they have no conflict of interest.

### Acknowledgments

The authors thank Prof. Guancong Ma for fruitful discussions. This work was supported by the National Key R&D Program of China (2018YFA0306200 and 2017YFA0303702), the National Nat-

ural Science Foundation of China (11890700 and 11625418), and the Natural Science Foundation of Guangdong Province (2020A1515010634).

### Author contributions

Ze-Guo Chen and Minghui Lu conceived the idea. Ze-Guo Chen, Wei Wang, Chudong Xu, and Taotao Zheng built the model. Chudong Xu and Hao Ge performed experimental measurements. Ze-Guo Chen, Ming-Hui Lu, and Yan-Feng Chen guided the research. Chudong Xu, Ze-Guo Chen, Taotao Zheng, and Ming-Hui Lu wrote the paper. All authors contributed to discussions of the results and paper preparation.

### Appendix A. Supplementary materials

Supplementary materials to this short communication can be found online at <https://doi.org/10.1016/j.scib.2022.09.002>.

### References

- [1] Hasan MZ, Kane CL. Colloquium: topological insulators. *Rev Mod Phys* 2010;82:3045–67.
- [2] Rechtsman MC, Zeuner JM, Plotnik Y, et al. Photonic floquet topological insulators. *Nature* 2013;496:196–200.
- [3] Chen H, Yao YL, Nassar H, et al. Mechanical quantum hall effect in time-modulated elastic materials. *Phys Rev Appl* 2019;11:044029.
- [4] Thouless DJ. Quantization of particle transport. *Phys Rev B* 1983;27:6083–7.
- [5] Nakajima S, Tomita T, Taie S, et al. Topological Thouless pumping of ultracold fermions. *Nat Phys* 2016;12:296–300.
- [6] Cheng W, Prodan E, Prodan C. Experimental demonstration of dynamic topological pumping across incommensurate bilayered acoustic metamaterials. *Phys Rev Lett* 2020;125:224301.
- [7] Zilberberg O, Huang S, Guglielmon J, et al. Photonic topological boundary pumping as a probe of 4D quantum hall physics. *Nature* 2018;553:59–62.
- [8] Lustig E, Weimann S, Plotnik Y, et al. Photonic topological insulator in synthetic dimensions. *Nature* 2019;567:356–60.
- [9] Rosa MIN, Pal RK, Arruda JRF, et al. Edge states and topological pumping in spatially modulated elastic lattices. *Phys Rev Lett* 2019;123:034301.
- [10] Riva E, Rosa M, Ruzzene M. Edge states and topological pumping in stiffness-modulated elastic plates. *Phys Rev B* 2020;101:094307.
- [11] Chen H, Zhang H, Wu Q, et al. Creating synthetic spaces for higher-order topological sound transport. *Nat Commun* 2021;12:5028.
- [12] Chen Z, Zhu W, Tan Y, et al. Acoustic realization of a four-dimensional higher-order chern insulator and boundary-modes engineering. *Phys Rev X* 2021;11:011016.
- [13] Fedorova Z, Qiu H, Linden S, et al. Observation of topological transport quantization by dissipation in fast Thouless pumps. *Nat Commun* 2020;11:3758.
- [14] Lu L, Gao H, Wang Z. Topological one-way fiber of second chern number. *Nat Commun* 2018;9:5384.
- [15] Fan X, Xia T, Qiu H, et al. Tracking valley topology with synthetic weyl paths. *Phys Rev Lett* 2022;128:216403.



Chudong Xu received his Ph.D. degree in Physics from Sun Yat-sen University in 2012. From 2014 to 2016, he was a postdoctoral fellow of the University of York. He is currently an associate professor at the College of Electronic Engineering, South China Agricultural University. His working fields include topological acoustic metamaterial and ultra-fast magnetization dynamics.



Ze-Guo Chen is an assistant professor at the Institute of Functional Materials and Intelligent Manufacturing, Nanjing University. He received his Ph.D. degree in Applied Mathematics and Computer Science from King Abdullah University of Science and Technology (2018). His research interest is metamaterials and topological physics.



Ming-Hui Lu received his Ph.D. degree from Nanjing University in 2007. From 2011 to 2012, he was a visiting scholar of Stanford Institute for Materials & Energy Science of Stanford University. He is currently a professor at the College of Engineering and Applied Sciences, Nanjing University. His current research mainly focuses on fundamental study of photonic and acoustic artificial structures and metamaterials as well as their related applications.