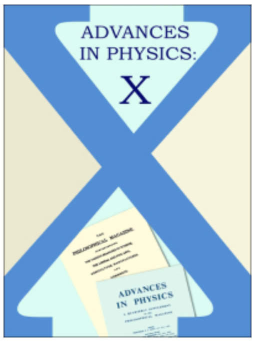

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REVIEW

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Optical simulation of various phenomena in curved space on photonic chips

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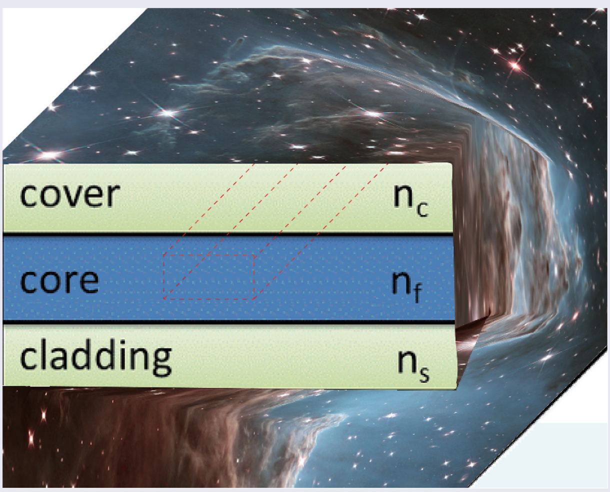
ABSTRACT
Transformation optics have been an essential paradigm to manipulate electromagnetic waves on the subwavelength scale and have brought various functional photonic architectures into integrated photonic chips. On the other hand, in the spirit of analogical thinking, classical and quantum simulations of general relativity have been extensively studied in diverse physical systems. In this review, we summarize recent advances in analogical gravitation based on integrated photonic chips with the aid of transformation optics. Meanwhile, different types of transformation optical structures, such as gradient waveguides, metasurface waveguides, waveguides on curved space and gradient waveguide arrays, emulating a variety of phenomena in curved space are reviewed, including the gravitational lensing of black holes, Einstein rings, cosmic strings, the particle pair evolution near the event horizon and so on. Furthermore, perspectives for the study of analogical gravitation based on integrated photonic chips are discussed.



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1. Introduction

Analogy has been widely used in scientific research and dates back to the ancient Greek philosopher Aristotle, who was the first to lay the foundations of the theory of analogical reasoning. Analogical study, which allows scientists to go beyond their own professional scope to conduct interdisciplinary research, has been assumed to be one of the most vital and efficient tools to explore and understand nature. For instance, De Broglie proposed the genuine hypothesis on wave properties of material particles inspired by the analogy between geometric optics and Newtonian particle dynamics, expressed by the formal similarity between variational Fermat's principle of ray optics and the least action principle of Newtonian mechanics. Another celebrated example is that Ernest Rutherford envisioned the atom in the spirit of the solar system, with electrons orbiting around a massive nucleus that occupy only a very small part of the atom. Moreover, it is worth mentioning that another notable analog case is topological physics [1,2], a current hot topic research, which originates from the concept of the topology in mathematics that deals with highly robust conserved quantities that do not change when physical objects are continuously deformed. Furthermore, topological phases, originally discovered in solid-state electron systems, have been expanded to other subfields of physics, such as optics and photonics [3,4], acoustic and mechanical systems [5,6].

In the spirit of analogical thinking, there has also been a fruitful intertwining between different branches of classical physical systems. Taking the early development of electrodynamics as an example, electric fields, magnetic fields and their laws are analogous to flow velocity fields and their laws, such as the introduction of concepts of vortex, source, sink, etc. Such analogues are essential to advance the development of electromagnetism. Additionally, another intriguing case is laboratory analogues of general relativity (GR). In the early 1920s, Gordon noticed that moving isotropic media appear to electromagnetic fields as certain effective space-time geometries [7]. In 1960, Plebanski formulated the analogy [8] between the macroscopic Maxwell's equations in complex inhomogeneous media and free-space Maxwell's equations for the background of an arbitrary spacetime metric, and especially proposed an original viewpoint that smooth deformation of space can be physically equivalent to inhomogeneous and anisotropic electromagnetic medium. Unfortunately, there are physical limitations to obtain such specially designed electromagnetic parameters for conventional materials made of atoms. In 2006, Pendry and Leondart independently and originally put forward transformation optics [9,10], which offers unparalleled opportunities for controlling light propagation through careful refractive index engineering using metamaterials. In particular, metamaterials [11], artificially engineered materials on the subwavelength scale, can be designed to induce customized properties with arbitrarily effective parameters that originally did not exist, such as the negative refractive material [12] and perfect imaging [13–15]. One of the amazing applications of transformation optics is invisibility cloaks [16–22], in which light is regarded as linear parallel geodesic rays in deformed spaces. Moreover, with the aid of transformation optics and metamaterials, various well-known phenomena related to GR have been emulated, ranging from artificial black holes [23–26], Minkowski spacetimes [27], electromagnetic wormholes [28], and De Sitter space [29] to cosmological inflation and redshift [30,31].

On the other hand, there has also been increasing interest in the analogy between classical physics and quantum physics. For example, based on the similarity between the paraxial optical wave equation describing the spatial propagation of a monochromatic light beam and the temporal Schrodinger equation for a quantum particle, a wide variety of coherent quantum effects encountered in atomic, molecular or condensed-matter physics have been mapped into optical waveguides. Given that understanding and accessing coherent wave phenomena in microscopic quantum systems is often a challenging task because of complications arising from many-body effects, decoherence, and the presence of time-dependent or nonlinear terms in the Schrodinger equation, such analogies in optical waveguides have the possibility of exploring coherent dynamical regimes not yet accessible in original quantum systems. For example, photonic lattices made of single-mode coupled waveguides have become a popular tool for simulating a plethora of quantum phenomena in condensed-matter systems, ranging from photonic Floquet topological insulators [32] and Weyl physics [33] using spiral waveguides, Landau levels using strain photonic lattices [34], Aharonov-Bohm cages [35] to the demonstration of flat-band states and dispersion-less light propagation [36]. At the same time, such quantum-classical analogies in waveguides in turn promote the acquisition of novel integrated optical functions and devices, such as fractional discrete Fourier transforms [37] and quantum logic gates [38] on such photonic chips. It is emphasized that such quantum-optical analogies using optical waveguides have been well summarized, and several pertinent reviews have been given [39,40]. In addition to quantum-optical analogies, other quantum-classical analog systems have also been demonstrated, for example, exploiting water surface waves to emulate Hawking radiation [41] and Penrose superradiance [42], employing electric circuit networks [43] to study higher-order skin-topological effects in condensed matter.

At the same time, the over past decades have witnessed remarkable progress in quantum simulations, Feynman's innovative idea [44] that one could employ a controllable quantum system to study another less controllable or inaccessible quantum system. A variety of quantum architectures

[45], such as atoms and ions, nuclear and electronic spins, superconducting circuits, and photons, with salient advantages of a high degree of controllability, novel detection possibilities and extreme physical parameter regimes, have been demonstrated as flexible quantum simulators for disposing of intractable problems in, e.g. condensed-matter physics [46], high-energy physics [47], quantum chemistry [48], and cosmology [49]. For instance, Hawking radiation [50], an undetectable quantum effect near the event horizon of black holes owing to the extremely low radiation temperature and lack of astronomical observation tools, has been successfully emulated in various quantum simulators, ranging from ultracold atoms [51], Bose–Einstein condensates [52], trapped ions [53], and Fermi-degenerate liquids [54] to superconducting circuits [55] and nonlinear optical media [56,57]. Additionally, the complex interacting many-body problems in natural condensed-matter physics, such as the Hubbard model [58,59], spin glasses [60], quantum phase transitions [61] and so on, have also been studied using quantum simulations with high efficiency. Encouragingly, what is more amazing is the recent advances of special quantum simulators for addressing certain mathematical tasks, such as the Sycamore quantum processor [62], Gaussian boson sampling machine *Jiuzhang* [63] and *Zuchongzhi* quantum processor [64], which have fully exhibited quantum advantages to perform well-defined intractable problems compared to classical computers.

As mentioned above, the classic and quantum simulations have cross-penetrated in different branches of physics and even various subjects of science, and have gained numerous and fruitful achievements with well-demonstrated vigorous vitality. For the sake of brevity, the aim of this view, hence, is not intended to cover all the related advances but to focus on the current progress on optical simulations of various classical and quantum phenomena about GR emulated on photonic chips using photons. Stirred by metamaterials and transformation optics, we introduce several concrete optical architectures on photonic chips to emulate some interesting classic and quantum phenomena related to GR, aim for a comprehensive treatment that incorporates state-of-the-art experiments, and especially point the way toward potential applications. This review is organized as follows: In Section 2 we briefly review the underlying principle of optical simulation of GR. In Section 3, we briefly classify optical waveguides on chips, which are judiciously designed and leveraged to carry on the research of optical simulation of GR. In Section 4, the optical simulation of classical GR phenomena using various types of photonic chips are reviewed, such as gradient waveguides, metasurface waveguides and waveguides on curved space. In Section 5, optical simulations of phenomena about quantum fields in curved space used gradient waveguide arrays are highlighted. In

Section 6, we summarize the outlook and future of optical simulation and quantum simulation on photonic chips.

2. The underlying principle of the optical simulation of GR

First, we briefly review the principle underlying the GR-optical analogies, which is based on the form invariance of Maxwell's equations between the complex inhomogeneous media and the background of an arbitrary space-time metric. It is emphasized that the specific description of the invariance has been well summarized in [8,25,65]. In general, a geometry of the space-time is described by the metric $ds^2 = \sum_{\alpha\beta} g_{\alpha\beta} dx^\alpha dx^\beta$, where we take x^α with Greek indices running from 0 to 3 to denote the space-time coordinates. Latin indices indicate the spatial coordinates and run from 1 to 3, whereas $x^0 = ct$ describes time measured in spatial units with c being the speed of light in vacuum. The metric tensor $g_{\alpha\beta}$ may vary as a function of the coordinates, because space-time may be curved or because curved coordinates are used in flat space. The determinant g of $g_{\alpha\beta}$ measures the four-dimensional volume of an infinitesimal space-time element. According to customary in general relativity, we denote the matrix inverse of $g_{\alpha\beta}$ by $g^{\alpha\beta}$, where the position of the indices indicates that $g_{\alpha\beta}$ is co-variant and $g^{\alpha\beta}$ is contra-variant under coordinate transformations. We give the free-space Maxwell equations in Cartesian components:

$$\sum_i \frac{\partial D^i}{\partial x^i} = \rho, \qquad \sum_i \frac{\partial B^i}{\partial x^i} = 0 \tag{1}$$

$$\sum_{jk} \in^{ijk} \frac{\partial H_k}{\partial x^j} = \frac{\partial D^i}{\partial t} + j^i, \sum_{jk} \in^{ijk} \frac{\partial E_k}{\partial x^j} = -\frac{\partial B^i}{\partial t} \tag{2}$$

where $\in^{ijk}$ is the completely antisymmetric Levi-Civita tensor. The spatial indices indicate that in this representation E_i and H_i form the components of vectors that are co-variant under purely spatial transformations, whereas D^i and B^i constitute contra-variant spatial vectors. The charge density ρ and the current density j are given by $\sqrt{-g} j^0$ and $c\sqrt{-g} j^i$ of the four-current j^α. In empty but possibly curved space, the electromagnetic fields are connected by the constitutive equations in SI units.

$$\boldsymbol{D} = \varepsilon_0 \varepsilon \boldsymbol{E} + \frac{\boldsymbol{w}}{c} \times \boldsymbol{H}, \boldsymbol{B} = \mu_0 \mu \boldsymbol{H} - \frac{\boldsymbol{w}}{c} \times \boldsymbol{E} \tag{3}$$

with the symmetric matrices ε and μ and the vector $\mathbf{w}$ given as

$$\varepsilon = \mu = -\frac{\sqrt{-g}}{g_{00}} g^{ij}, w_i = \frac{g_{0i}}{g_{00}} \tag{4}$$

We can see that empty but curved space-time manifolds or curved coordinates appear as effective dielectric media with anisotropic parameters ε and μ, especially with a magneto-electric coupling vector $\boldsymbol{w}$. Since the material representation equation (4) requires both electric and magnetic response and the magneto-electric coupling parameter, it is a huge challenge to realize these designed material parameters when the corresponding relation was firstly proposed. Fortunately, with the advent of metamaterials, the material parameters corresponding to some metrics of space-time have been realized.

Among these complex metrics of space-time, an isotropic central symmetric diagonal metrics has attracted attention thanking to easy experimental realization. For a class of centrally symmetric metrics $ds^2 = g_{00}(r')dt^2 - g_{r'r'}(r')dr'^2 - r'^2 d\Omega^2$ that can be written under coordinate transform in an isotropic form $ds^2 = g_{00}(r'(r))dt^2 - g(r)\left(dr^2 + r^2 d\Omega^2\right)dr'^2$ where $g(r) = [r'(r)/r]^2$, and $r = r(r\prime) = \exp \sqrt{g_{r\prime r\prime}} d\ln r\prime$. Note that both coordinate systems represent the same space-time. This equivalence allows the rendering of the material response to be reduced from anisotropic equation (4) to isotropic with an effective refraction index:

$$n(r) = \sqrt{g(r)/g_{00}(r)} = \frac{r'(r)}{r} \frac{1}{\sqrt{g_{00}(r'(r))}} \tag{5}$$

that could be achieved with pure dielectric non-dissipative and non-dispersive materials.

3. The classification of optical waveguide on chips for optical simulations

As is well known, photons, as information carriers, play an important role in the current information society. High efficiency manipulation of electromagnetic waves or photons on chips has always been a pursued goal in the optical community. In particular, the high development of the current information society requires increasingly advanced photonic integration technology [66]. Fortunately, a range of integrated photonics platforms, e.g. silica-on-insulator [67], laser-writing silica [68], silicon nitride [69], lithium niobite-on-insulator [70], gallium arsenide [71], indium phosphide [72] and many others, have exhibited a strong capacity for the manipulation of photons with high efficiency to obtain the desired effective electromagnetic parameters.

In general, optical waveguides are fundamental elements in integrated optics. Essentially, all integrated devices are built on waveguides. To some extent, the properties of optical waveguides determine the performances of integrated components on the chip. Structural design and refractive engineering for optical waveguides, thus, are important for the realization of novel and high-performance devices. On the other hand, recent years have seen a boom in metamaterial research. Hence, there is also surging motivation to bring metamaterials into optical waveguide technologies and on-chip architectures. This method has new degrees of freedom to control the flow of light in integrated photonic devices. In particular, with the aid of ever-improving nanofabrication techniques, such as electron beam lithography, focused ion beam, and deep ultraviolet lithography, subwavelength metamaterial structures can be fabricated in the same lithography step as conventional waveguides by using manufacturing processes that are well established in the semiconductor electronics industry, thus making their integration straightforward. A plethora of advanced integrated devices on chips with unprecedented performance have been well demonstrated, e.g. polarization beam splitters [73] and rotators [74,75], waveguide crossings and bends [76,77], mode converts [78] and multiplexers [79,80]. At the same time, the emerging concept of transformation optics, a design method for controlling photons at our discretion, opens up unique possibilities for advancing the integration of complex functionalities in photonic circuits, such as a flattened Luneburg lens [81], Maxwell's fish-eye lens [82] and dual-function 'Janus' devices [83].

Optical waveguides can be classified according to their geometry, mode structure, refractive index distribution, and material (see Figure 1). In general, optical waveguides comprise a longitudinally extended high-index medium called the Core, which are transversely surrounded by a low-index medium called the Cladding. In the following, invoked by the design concept of transformation optics and the mature development of fabrication at the micro-nanoscale, we can judiciously manipulate either the core layer or cladding layer or both in optical waveguides to realize distinct optical functions. Intriguingly, with the aid of nontrivial optical waveguides, we can investigate the large-scale gravitational astronomical phenomena predicted by GR, which are either inaccessible or cannot be studied repeatedly and carefully. On the other hand, it is noted that such devices inspired by GR can have unique advantages in some aspects compared to conventional optical devices.

4. Optical simulation of classical GR phenomena

Gradient waveguides

Recently, gradient waveguides whose core thickness varies at our discretion have attracted considerable attention in the manipulation of photons on

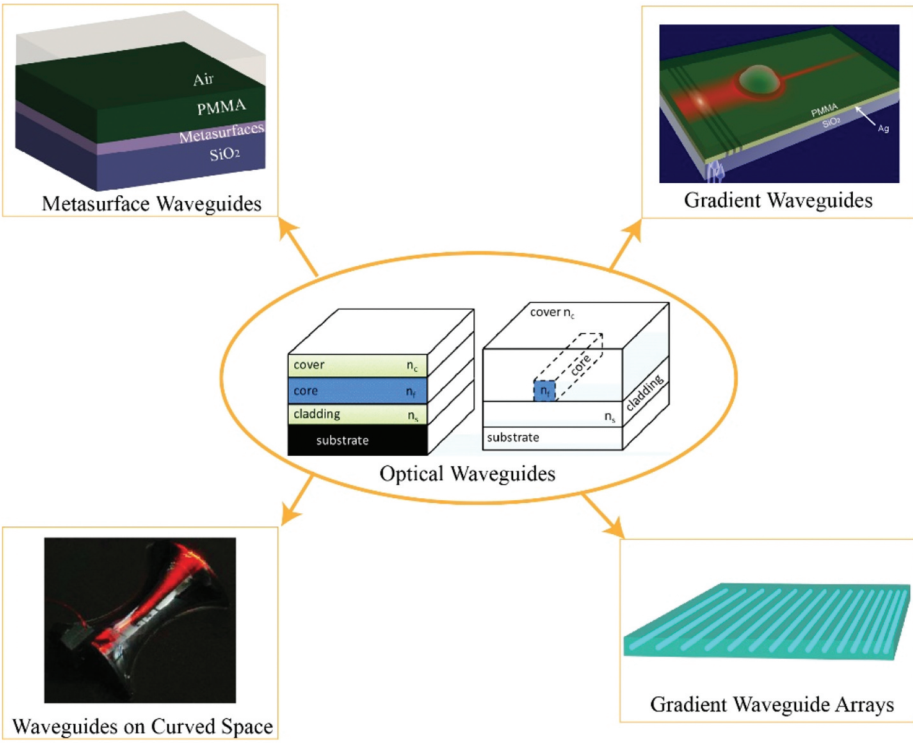

Figure 1. Different type waveguides for optical simulations inspired by general relativity.

integrated chips. For instance, Thomas Zentgraf et al [81]. explored grey-scale lithography to adiabatically tailor the topology of a dielectric layer adjacent to a metal surface to demonstrate a plasmonic Luneburg lens and Eaton lens that can, respectively, focus and bend surface plasmon polaritons (SPPs). Shuyi Li et al [82]. also used greyscale electron-beam lithography to adiabatically change the profile of a Si layer on a commercial silicon-on-insulator wafer to achieve a Maxwell fisheye lens for multimode waveguide crossing, which has manifested an advantage in on-chip multimode routing and communications with a highly integrated and large capacity. Hyuck Choo et al [84]. experimentally demonstrated the achievement of highly efficient nanofocusing of electromagnetic waves into the deep subwavelength regime by leveraging a three-dimensional Au–SiO_2–Au taper with a carefully engineered SiO_2 layer. Xiangyang Wang et al [85]. exploited a gradient polymer waveguide to achieve the Mikaelian lens to simultaneously obtain a self-focusing effect in the geometric-optics limit and a Talbot effect in the wave-optics limit. In particular, the Talbot effect constructed in such a lens can be used to transfer a digital field pattern without diffraction and has potential applications in digital coding communications. Additionally, optical cloaking [86] and optical super-imaging [87] have also been achieved with the aid of gradient waveguides.

Interestingly, gradient waveguides with high efficiency to model light can be utilized to mimic large-scale astrophysical phenomena in curved spacetime, such as celestial mechanics, Einstein's field equations, gravitational lensing and the event horizons of black holes. Chong Sheng et al [88]. judiciously embedded a microsphere into a planar polymer waveguide formed during a controlled spin-coating process (see Figure 2 (b)). Due to surface tension effects, the polymer waveguide layer around the microsphere is distorted, resulting in a continuous change in the waveguide effective refractive index that, under certain conditions, can mimic the curved spacetimes caused by strong gravitational fields. In such a gradient waveguide, the lensing and asymptotic capture of the incident light in an unstable circular orbit that corresponds to the photon sphere of a compact stellar object are observed (see Figure 2 (c, d)). Following the same idea, the gradient waveguide can also demonstrate Einstein's Rings [89], which is a famous phenomenon predicted by GR and observed in astronomy. The Einstein ring phenomenon occurs when light from a point source is deformed by a mass distribution through gravitation lensing that causes the appearance of a ring around the mass distribution. In such a setting, the spherical wavefront produced by the arc grating emulates the wave radiated outwards from a point source. When this 1D spherical wavefront passes by the star, it is focused, and the beam width changes as a function of the propagation distance, as extracted from the experimental data. Typical results for two different 'stars' (microdroplets with two different radii) are fabricated, as shown in Figure 2 (g, h). As the radius of the 'star' is larger, the convergence of the beam is more extreme, but the final beam is wider. At this point, it is intriguing to compare emulation results with Einstein's prediction. The Einstein formula for the angular diameter of the virtual ring $\beta = \sqrt{\alpha_0 R_0/z}$ depends on the convergence angle α_0, the radius of the mass distribution R_0 and the distance between the center of the mass distribution to the observation point z. To conform with the Einstein formula, the relative angular radius between the two mass distributions (two samples) was calculated and agreed well with theory (see Figure 2 (i)). Aside from gravitational lensing, Igor I. Smolyaninov theoretically proposed to utilize a gradient waveguide for emulating the Unruh effect [90,91], that is, an accelerating observer will treat its surrounding environment as a thermal radiation bath (see Figure 2 (j)). In this theory, when photons launched into a specially designed optical gradient waveguide will act as massive quasi-particles and experience strong acceleration, such a waveguide can be used as a thermometer that would measure the Unruh temperature. Subsequently, Hui Ge et al. exploited a gradient optical fiber [92] to validate this Unruh effect using a leaky phenomenon within the optical fibers akin to bremsstrahlung (see Figure 2 (l)). Additionally, the emulated curved space in gradient waveguides can also be utilized as a tool

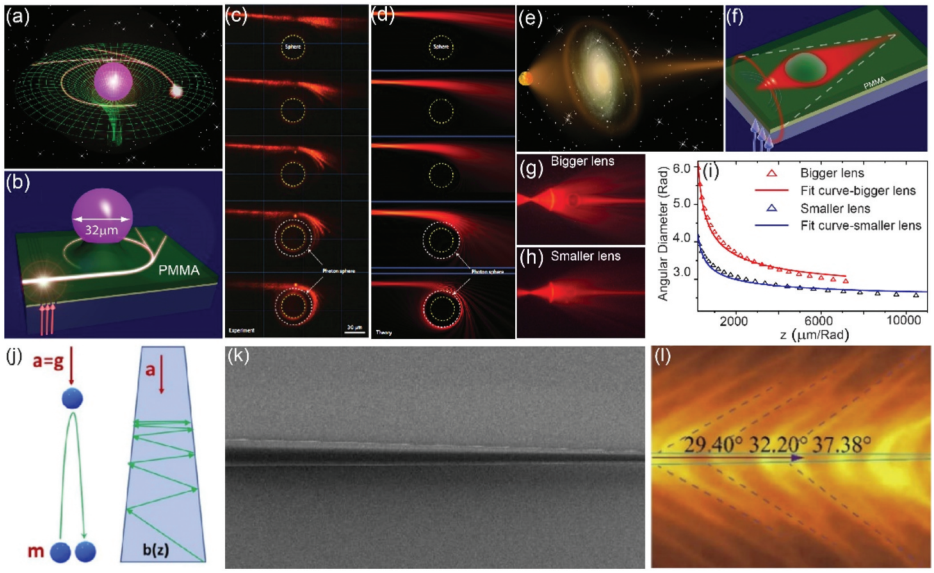

Figure 2. Gradient waveguides for emulating various general relativity phenomena. (a)-(d) Gravitational lensing of artificial black holes [88]. (a) Depiction of light deflection by the gravitational field of a massive stellar object. (b) Schematic view of the microstructured optical waveguide formed around a microsphere and used to emulate the deflection of light by a gravitational field. (c, d) Scattered field intensity observed in the experiment and calculated using a full-wave finite-difference frequency-domain (FDFD) electromagnetic code. (e)-(i) Einstein rings [89]. (e) Einstein's vision: light from a point source is focused by a gravitational lens, and is subsequently observed as a virtual ring around the mass distribution. (f) Schematic view of the fabricated inhomogeneous waveguide. (g, h) Experimental results showing a spherical wave passing through two domes with different radii. (i) Fit to Einstein's formula for the angular diameter of the Einstein rings. (j) Unruh effect in tapered waveguides [91]. (j) Massive body thrown up in a gravitational field that can be emulated by waveguide tapering. (k)-(l) The bremsstrahlung in a tapered optical fiber [92]. (k) A tapered fiber. (l) The radiation phenomenon generated by a tapered fiber.

of wavefront shaping in waveguide settings. For instance, the transformation of a broad Gaussian beam (FWHM $\sim 30\mu m$) into a very narrow nondiffracting beam (FWHM $\sim 2\mu m$) with the propagation of $200\mu m$ was achieved in a predesigned gradient waveguide by drawing on intuition from GR [89]. A gradient waveguide whose effective index satisfies a gravitational field profile in the form of a linear potential could result in the oscillation between an incident broad Gaussian beam and an accelerating Airy beam [93].

Metasurfae waveguides

Metasurfaces, two-dimensional metamaterials, have been important tools to control electromagnetic waves in both far-field and near-field regimes. Compared with a three-dimensional bulky metamaterial, this artificial plane material with subwavelength thickness greatly reduces fabrication time and mitigates fabrication complexity. Additionally, traditional metamaterials usually control the wavefront of an electromagnetic wave by accumulating the phase by propagating at a distance far larger than the wavelength. However, a metasurface can efficiently manipulate the wavefront of an incident electromagnetic wave through just the subwavelength propagation distance. Therefore, this can largely alleviate the propagation loss. A more detailed discussion of remarkable progress in metasurfaces for controlling electromagnetic waves on integrated chips have been well summarized in reference [94,95].

Interestingly, based on the high efficiency of controlling light by using metasurfaces, a metasurface waveguide can be utilized to conduct research on analogical gravitation. Theorists predicted that some topological defects may have formed during a symmetry breaking of the Higgs vacuum field in the early universe when the topology of the vacuum manifold associated with this symmetry breaking was not simply connected. Topological defects of spacetime include 0-dimensional monopoles, 1-dimensional cosmic strings and 2-dimensional domain walls. Fortunately, stimulated by the discovery of gravitational waves [96,97], a stochastic gravitational wave background possibly emitted by a cosmic-string network [98,99] and cosmic domain walls [100] generated in the early Universe has been reported. Aside from gravitational waves, many other interesting nontrivial phenomena about these cosmic topological defects [101–108] also have been predicted. Interestingly, C. Sheng et al [109]. experimentally demonstrated definite photon deflections caused by emulating cosmic strings with the aid of metasurface waveguides, whose structure combines a slab waveguide with a rotational metasurface (see Figure 3(a,e)). Because cosmic strings can change the topological structure of surrounding space-time to result in the defect or surplus of space-time angle, when light propagates in such a topological space-time, it will produce a definite deflection angle $\Delta\theta = 4\pi G\mu/(1 - 4G\mu)$ (G is the gravitational constant, μ is the linear mass density of the cosmic string), which is only determined by mass density parameter μ that originates from the intrinsic topological properties of the cosmic string, and which is regardless of the incident position, propagation direction, wavelength and polarization direction of photons. If $\mu < 0$, the string has a positive mass density and carries positive curvature at the origin, and the photon undergoes an attractive force and moves toward the origin of the string. When $\mu > 0$, the string has a negative mass density and carries negative curvature at the origin, and the photon undergoes a repulsive force and moves away from the origin of the string. By judiciously controlling the parameters of rotational metasurfaces, robust definite photon deflection in both positive and negative topological space has been observed

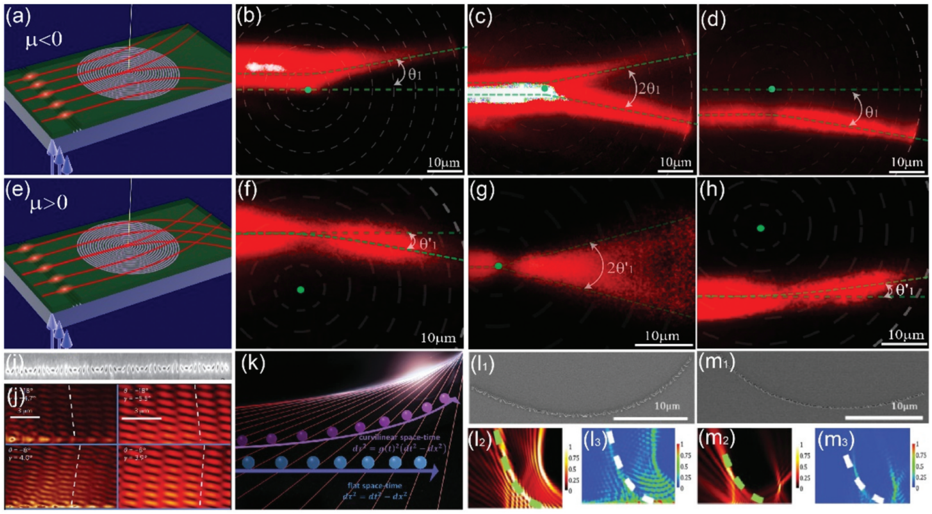

Figure 3. Metasurface waveguides for emulating various general relativity phenomena. (a)-(h) Definite photon deflections by cosmic stings [109]. (a, e) The schematic of cosmic string with negative ($\mu < 0$) and positive ($\mu > 0$) mass density using metasurface waveguides. (b)-(d) Photon deflection observed in the experiment under different impact parameters with negative mass density. (f)-(h) Photon deflection observed in the experiment under different impact parameters with positive mass density. (i, j) Cherenkov surface plasmon wakes [110]. (i) Scanning electron micrograph of the nanostructures used to generate the SPP wakes. (j) Near-field scanning optical microscopy images and the calculated results. (k-m) The Bremsstrahlung radiation of moving particles using metsurfaces [111]. (k) Geometric picture describing the same event in different coordinates through mimicking Bremsstrahlung radiation of moving particles (blue ball and purple ball) in flat space-time and curvilinear space-time respectively. (l1, m1) SEM images of different samples. (l2, l3) The simulations and experiments for the case of l1. (m2, m3) The simulations and experiments for the case of m1.

as shown in Figure 3(a-h). This remarkably robust photon deflection can be used as a new kind of omnidirectional lens, which can bend light without changing the beam profiles. Such a unique property can be used in optical focusing, imaging, and information transfer.

In addition to cosmic strings, the accelerating particles in general relativity can be well-emulated by leveraging a one-dimensional metasurface that generates a predefined caustic of SPPs on a metal surface. Genevet et al [110]. utilized SPPs generated on a metasurface through spin-orbit interaction as an analogous system to study Cherenkov radiation from a charged particle moving with a constant high speed. F. Zhong et al [111]. also explored the geometric phase of SPPs caused by the spin-orbit interaction of photons to emulate accelerating frames in general relativity and obtained a new kind of accelerating beam, the Rindler beam, based on the Rindler metric in gravity. Moreover, compared to metamaterials with the limited scope of the effective index, the geometric phase of SPPs with the aid of metasurfaces provides a freedom to control the effective index that satisfies the requirement of the large effective index for emulating curved space.

Waveguides on curved space

In contrast to engineering space-dependent electric and magnetic parameters using metamaterials or metasurface to observe curved-space phenomena in the laboratory, another route for such studies is to create curved space by engineering the geometry of the space itself. This idea [112], suggested in 1981, started by exploring the dynamics of a free quantum particle rigidly bounded to a curved surface, resulting in the Schrodinger equation containing a linear potential that is a function of the curvature.

Likewise, these ideas were brought to electromagnetic waves and first demonstrated by Batz and Peschel [113], who explored the dynamics of light propagating within a thin-film waveguide covering the curved surface area of a three-dimensional body, effectively creating 2D curved space for the light. These works [113,114] clearly show that when a Gaussian beam propagates on a surface with constant positive curvature, periodic refocusing, self-imaging, and diffractionless propagation can be observed as shown in Figure 4(a). In contrast, light spreads exponentially on surfaces with constant negative Gaussian curvature (see Figure 4(b)). Furthermore, the evolution of speckle patterns propagating along two-dimensional surfaces of constant positive and negative Gaussian curvature are utilized to study the influence of intrinsic curvature on HBT measurements [115]. In addition to Gaussian beams, shape-preserving accelerating beams propagating on a curved space [116,117] with a constant positive curvature have been studies, which found that unlike accelerating beams in flat space, these wave packets change their acceleration trajectory due to the interplay between interference effects and the space curvature, and focus and defocus periodically due to the spatial curvature of the medium in which they propagate (see Figure 4(c)). Additionally, C.N. Xu et al [118]. theoretically studied the Gouy phase of light beams in both the paraxial and nonparaxial regimes on a two-dimensional curved surface by generalizing the angular spectrum method and found that there exists an extra phase shift introduced by the curvature of the surface. G.H. Liang et.al devised an air helicoid waveguide [119] which was obtained with an air layer sandwiched by two metal helicoid surfaces to mimic an expanding universe. W.F. Ding et al [120]. utilized a curved space to study laser propagation in a Rindler accelerated reference frame based on matrix optics.

More interestingly, such waveguides on curved space have been extensively utilized to emulate some general relativity phenomena about black holes and wormholes. R. Bekenstein et al [121]. investigated the evolution of light in a paraboloid structure inspired by the Schwarzschild metric

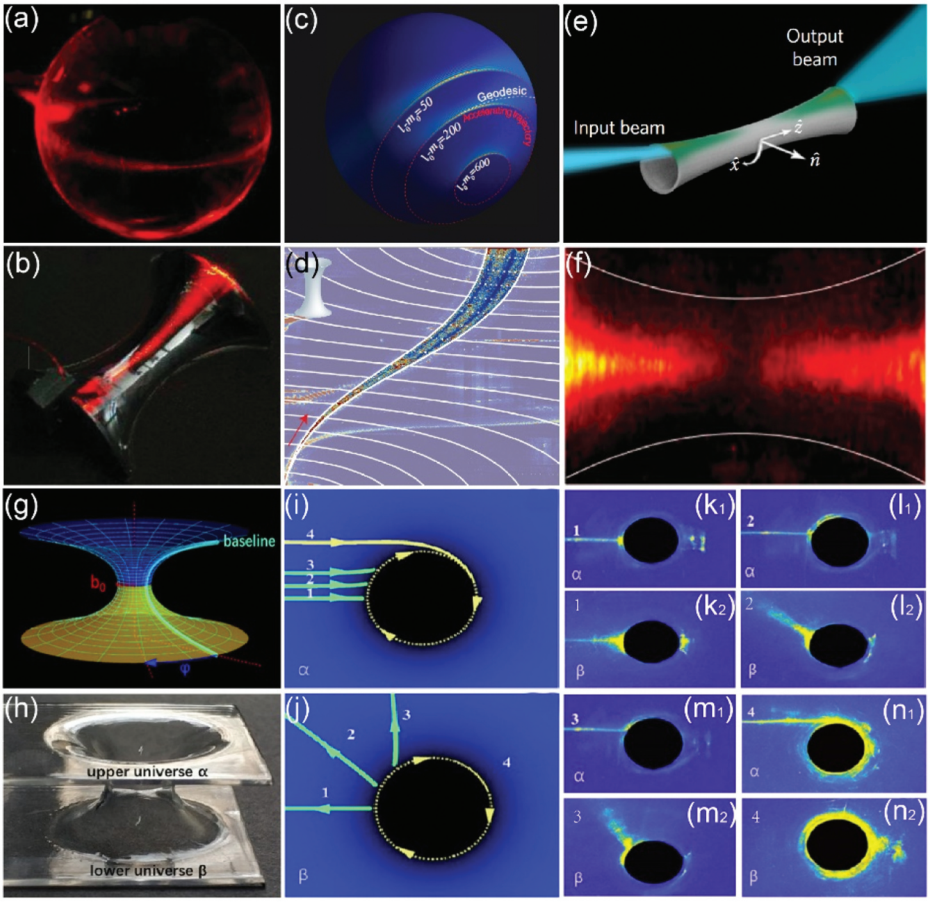

Figure 4. Waveguides on curved space for emulating various general relativity phenomena. (a, b) An initial Gaussian beam on a positively and negatively curved surface [114]. (a) Experimental realizations of light propagation on a sphere with a self-focusing. (b) Experimental realizations of light propagation on a hyperbolic with exponentially spreading. (c) Accelerating Wave Packets in Curved Space [117]. (d) The speckle evolution along the hourglass [115]. (e, f) Control of light by curved space in nanophotonic structures. (e) Schematic of the coupling scheme of the light to the paraboloid waveguide [121]. (f) Curvature effects on diffraction. (g-n) Simulations of wormhole using curved optical spaces [122]. (g) The embedding diagram of the 2D wormhole. (h) The image of the sample fabricated with 3D printing technique. (k-n) The calculated light geodesic lines with four impact parameters for the case that the light enter the wormhole through the on plane and exit through the other plane.

describing the space surrounding a massive black hole. In contrast to 2D fabrication techniques imported from microelectronics, this work employs the multiphoton polymerization technique to construct a special 3D waveguide whose size is comparable to the wavelength scale (Figure 4(e)). Such a curved nanophotonic structure can not only control the diffraction properties and the phase and group velocities of wavepackets propagating within it but also exhibit tunneling through an electromagnetic bottleneck (see

Figure 4(f)). In addition to the Schwarzschild metric of black holes, R. Q. He et al [122]. also employed a waveguide on curved space inspired by the Morris-Thorne traversable wormhole metric to show that a giant tidal force induces the divergence of optical deflection at the throat of the wormhole (see Figure 4(h)). In this work, they gradually changed the impact parameter P compared to the radius of the wormhole's radius b_0 and compared three different situations. (1) When $P/b_0 > 1$, the calculated light rays do not pass through the wormhole. In the process, the deflection angle ϕ increases with the decrease in P. (2) When $P/b_0 < 1$, all three rays pass through the wormhole. They enter the wormhole through the one plane and exit through the other plane (see Figure 4 (i-m)). (3) When $P/b_0 = 1$, the rays are trapped within the stable orbit at the throat of the wormhole (see Figure 4 (n)). Furthermore, considering the incident Gaussian beam with a certain width, the spread width of the wavepacket caused by the tidal force is obtained by comparing the divergence deflection angle for both edges of the incident beam. Additionally, similar wormholes have also been emulated in waveguides on curved space for elastic waves [123].

At the same time, waveguides on curved space have also inspired optical functional devices. One of the limitations of metamaterials that restricts us from exploring more complicated optical devices is the demand for either tremendous change or a singularity in the material parameter, which requires a high level of material fabrication. L. Xu et al [124]. utilized waveguides on curved space to emulate topological defects with a singularity and achieve bending optical devices with a constant angle. Moreover, geodesic lenses [125], which are well-known absolute instruments, were obtained by leveraging such waveguides on curved space.

5. Optical simulation of a quantum field in a curved space

Gradient waveguide arrays

Over the last decades, arrays of evanescently coupled waveguides have been brought into focus as a particular platform for manipulating electromagnetic waves, in which the dispersion and diffraction of propagating light can be specifically modified. Moreover, it turns out that the light evolution in these systems shares fundamental similarities to the quantum evolution of particle functions in the context of relativity, so that waveguide arrays can behave as a toy model system for emulating quantum fields related to GR. For instance, F. Dreisow et al [126]. exploited a one-dimensional binary waveguide array to simulate the *Zitterbewegung* (trembling motion) of a free Dirac electron, whose dispersion curves in the vicinity of the edges of the first Brillouin zone are mathematically equivalent to the typical hyperbolic energy-momentum dispersion relation for positive-energy and negative-energy branches of a freely moving relativistic massive particle. Likewise, massless Dirac particles [127], the vacuum instability and pair production [128,129], neutrino oscillations [130] and Majorana fermions [131] have also been well simulated in waveguide arrays in the spirit of the similar principle. However, these simulations belong to the realm of classic photonic simulation. It is appealing to consider the quantum aspect of analogical gravitation with the aid of quantum optics.

Fortunately, recent years have witnessed remarkable advances in quantum optics technology. Y. Wang et al [132]. proposed and experimentally demonstrated a quantum evolution of fermions in close proximity to an artificial black hole, which was emulated using bilayer waveguide arrays fabricated by a femtosecond laser direct writing technique (see Figure 5(a)). Due to the high controllability over the coupling and the on-site energy among waveguides, one-dimensional artificial black holes with the Schwarzschild metric can be constructed using evanescently coupled waveguide arrays with inhomogeneous hopping coefficients inspired by the concept of transformation optics. Compared to the linear time evolution in flat space, the dynamic behavior of a single-photon wave packet near the event horizon of the black hole has an exponential form over time, whose exponential index depends on the curvature of the black hole. More interestingly, the quantum evolution process of a single-photon wave packet in such predesigned waveguide arrays that emulate the event horizon of black hole exhibits a good analogue of Hawking Radiation: a single-photon wave packet with positive energy escapes from the black hole while negative energy is captured as shown in Figure 5(b). Moreover, it is an attractive question about the quantum evolution of entangled photons near the event horizon of black holes. C. Sheng et al [133]. theoretically studied the quantum walks of single photons, two indistinguishable photons and entangled photons in gradient waveguide arrays that emulate the event horizon of black holes. The optical trapping of single photons and two indistinguishable photons in such nonuniform lattices conforms to a well-known classical physical recognition due to the strong gravitational force of black holes (see Figure 5(c)). Counterintuitively, there is an optical escape for path-entangled photons for which one photon is captured, while the other photon escapes (Figure 5(d)). Intriguingly, the counterintuitive phenomenon has a distinct escape mechanism compared to Hawking radiation, which is wholly due to quantum interference. Moreover, entanglement entropy is also studied in such a toy model system, which clearly shows that the maximally entangled state becomes less entangled and monotonically decreases with the increasing curvature of black holes.

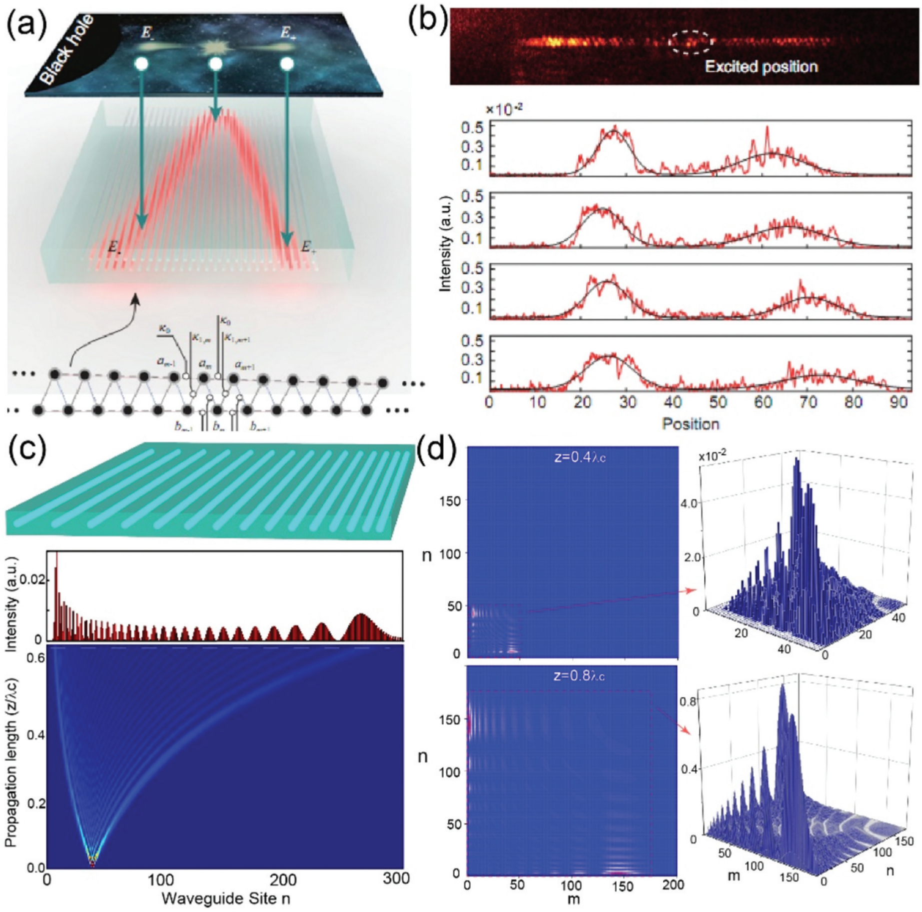

Figure 5. Quantum simulations of general relativity phenomena with waveguide arrays. (a, b) Quantum simulation of particle pair creation near the event horizon [132]. (a) The schematic of mapping the behavior of fermion pair into a waveguide array. (b) The imagined output probability distribution of single-photon wave packet and the output probability distributions with different excited positions in the same lattice. (c, d) Quantum walks of single photons and entangled photons in nonuniform waveguide arrays corresponding to a noninertial frame with the Rindler metric [133]. (c) Quantum walks of single photons exhibits that photons are captured by the event horizon. (d) Quantum walks of path-entangled photons exhibits that one photon is captured, while the other photon escapes.

6. Perspectives and conclusion

Over the years, integrated photonic chips have emerged as a well-controlled toy model system to study various large-scale gravitational astronomical phenomena in the tabletop environment. Although such analogies are just simulations, the underlying physical principle of these phenomena in both analogical systems and the real universe is supported by similar mathematical equations. Many phenomena predicted by GR are difficult to be observed and cannot be repeatedly studied. Thanks to analogical studies in the laboratory environment, one can accurately and repeatedly investigate these phenomena to verify the backing physics. Furthermore, the study of analog gravity based on photonic chips may address some more challenging works. For example, quantum gravity is a great challenge field in modern physics that seeks to describe gravity based on the principles of quantum physics and finally needs unification of quantum mechanics and general relativity. Hence, with the aid of the study of analogical gravitation based on integrated photonic chips, possible experimental works on quantum gravity will be carried out. It is noticed that a photon itself is a good quantum system and can be constructed into various types of quantum entangled states. Meanwhile, various types of curved space can be well-emulated on integrated photonic chips with the aid of metamaterials and transformation optics, such as De Sitter space and Anti-De Sitter space. The combination between the quantum entangle state and the emulated curved space on integrated photonic chips may bring much meaningful thinking to explore and understand the essence of gravity.

On the other hand, the study of analogical gravitation based on integrated photonic chips in turn has inspired various optical functional devices on chips, ranging from optical omnidirectional absorption cavities akin to black holes, nondiffractive optical beams inspired by the Einstein ring to definite nontrivial electromagnetic scatterings inspired by cosmic strings. It is noted that these gravitational inspired optical devices are achieved with the aid of the manipulation of the spatial dimension of gravitational field. The temporal dimension is another important and essential variable to describe curved spacetime. At the same time, spatiotemporal manipulation very recently has emerged as a new paradigm for wave manipulation using time-varying media [134]. Therefore, if we account for the temporal dimension in the design of optical devices on integrated chips, many more fascinating optical devices and applications, inspired by the study of analogical gravitation, may be achieved, such as the nonlinear frequency conversion and a reconfigurable quantum simulator.

In conclusion, on the one hand, the research of analogical gravitation on integrated photonic chips may promote the understanding of the basic physics of gravity. On the other hand, such studies inspire us to go beyond traditional photonic devices to bring new applications. We gave a brief overview, as well as perspective, of various GR phenomena based on the integrated photonic chip. With the aid of analogical thinking about gravity, we believe that tremendous new opportunities both in basic physics of gravity and optical devices inspired by gravity still wait to be explored.

Disclosure statement

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