

Topological Photonic States in Waveguide Arrays

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Topological photonics, accompanied by the ability to manipulate light, has emerged as a rapidly growing field of research. More platforms for displaying novel topological photonic states are being explored, thus offering efficient strategies for the realization of robust photonic devices. Optical waveguide arrays, described as a $(n+1)$ -dimensional system, are ideal platforms for studying topological photonics because of the characteristic that can exhibit light dynamics. Here, this work reviews the experimental implementations of the various topological phases in the optical waveguide arrays, and specifically discusses novel physical phenomena arising from the combination of topology with non-Hermiticity and nonlinearity. It is believed that topological waveguide arrays provide powerful support for enriching topological physics and promoting the development of topological photonic integrated devices.

1. Introduction

Topological insulators (TIs), characterized by topological order rather than relying upon traditional Landau–Ginzburg theory, describe a class of novel solids. One hallmark of the TIs is the existence of boundary states immune to backscattering, thus finding promising applications in quantum computation and low-power electronics.^[1–4] The research on TIs began with the discovery of the integer quantum Hall (QH) effect in 1980.^[5,6] In the QH effect, the bulk of a two-dimensional (2D) electron gas under a strong magnetic field is insulating, but the edge is conductive and exhibits robust and quantized Hall conductance.^[7] Thouless and Kohmoto related the quantized value to the system's topological invariant to describe the topological properties of the materials.^[8,9] The interface between two materials with different topological invariants hosts the localized states, and the current

can only unidirectionally propagate along the interface.^[10] This field became mainstream in condensed matter physics when the quantum spin Hall (QSH) effect was discovered in 2005.^[11] Unlike the QH effect requiring the breaking of time-reversal (TR) symmetry, the QSH system hosts topologically protected boundary states even though the TR symmetry is preserved.^[12] Since then, a large number of theoretical and experimental studies on topological states have been reported.^[13–24] In addition to the electric systems, the topological effects have been found achievable in photonics.

In 2008, Haldane and Raghu constructed analogs of QH edge modes in photonic crystals, and theoretically predicted that the electromagnetic wave in these modes is only allowed unidirectionally propagate along the edge.^[25,26] Wang experimentally confirmed the idea in the same year.^[27,28] They observed the unidirectional edge states at the interface between a magneto-optical and regular photonic crystal. These pioneering works spurred numerous subsequent investigations on photonic TIs.^[29–35] Numerous of the topological effects that exist in electronic systems could find their analogs in photonics, for example: QSH edge states emulated in the ring resonator arrays;^[36–38] photonic Floquet TIs implemented by applying temporal modulation;^[39–43] quantum valley Hall edge states realized in an all-Si hexagonal photonic crystal with broken inversion symmetry;^[44] higher-order photonic topological states demonstrated by using hexagonal and square lattices.^[45–47] The introduction of topology into photonics is revolutionary because it provides a robust way to control photons, which may yield many practical applications.^[48]

An ideal platform for studying topological photonics is waveguide arrays, and this is considered for three attractive reasons. First, the waveguide arrays provide a more convenient way to exhibit photonic topological phases. For instance, photonic Floquet TIs and valley Hall photonic TIs can be constructed by modulating the waveguide shape and the waveguide refraction index.^[39,49] Second, introducing gains and losses to systems in the spatial and temporal domains provides richer topological physics. In fact, the system with losses is more in line with the real system. Therefore, theoretical and experimental explorations of non-Hermitian topological structures are expected to promote the practical application of topological photonic devices.^[50,51] In contrast to the photonic crystal slabs, losses in the waveguide arrays are more accessible to control by wiggling the waveguide and introducing scattering points. Last, nonlinear effects in photonics are expected to unlock advanced functionalities of the topological system.^[52,53] Waveguide arrays may provide an effective carrier

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to explore nonlinear effects due to the intrinsic nonlinearity of the host medium, such as photorefractive crystals and niobate crystals. The advantages above have generated vast research interest in topological waveguide arrays in the past few years. However, topological waveguide arrays have been introduced only as a minor part of the existing reviews of topological photonics,^[54–59] making it difficult for readers to understand the topological effects in photonic waveguide lattices coherently and comprehensively.

Here, we review the significant experimental progress in the topological waveguide arrays, focusing on the photonic topological phases and their interactions with non-Hermiticity and nonlinearity. Section 2 begins with a basic introduction to topological optical waveguides, including the coupled mode theory and fabrication methods of the waveguide arrays. Section 3 reviews recent experimental results on 1D and 2D topological waveguide arrays. Various topological phases are distinguished. Section 4 reviews the topological effects of the non-Hermitian waveguide system, and displays some peculiar topological phenomena. In Section 5, the interplay between nonlinearity and topological phase in waveguide arrays is elaborated. We discuss the practical applications of topological waveguide arrays in Hermitian and non-Hermitian systems in Section 6. Finally, Section 7 concludes by providing an outlook on further advances and applications.

2. Foundation of Photonic Waveguide Array

2.1. Basic Theory

The waveguide is a common device for propagating electromagnetic waves, as well as an essential element in integrated optics. When multiple waveguides are arranged into an array, the single waveguide mode expands into a band due to the evanescent wave coupling between the waveguides. With the rise of topological ideas, the bands of waveguide arrays are linked with the topological phases. The topological boundary states can be constructed in waveguide arrays by adjusting the waveguide parameters. The theoretical description of the topological states in the waveguide arrays is based on coupled mode theory.^[60,61] Here, we start with classical Maxwell's wave equation. In a nonmagnetic medium, when a condition that the amplitude of the electromagnetic wave varies slowly in the propagation direction (z direction) is satisfied, the Helmholtz equation can be deduced as

$$\nabla_{\perp}^2 \varphi(x, y, z) + 2ik_z \frac{\partial}{\partial z} \varphi(x, y, z) - k_z^2 \varphi(x, y, z) + k_0^2 n^2(x, y) \varphi(x, y, z) = 0 \quad (1)$$

where $\varphi(x, y, z)$ denotes the envelope of the electric field, $\nabla_{\perp}$ operator acts on the transverse (x, y) plane. k_z represents the wave vector in the z direction and k_0 represents the wave number of free space, n is the refractive index. By formal transformation, Equation (1) can be written in a form analogous to Schrödinger's equation, as following

$$i \frac{\partial}{\partial z} \varphi(x, y, z) = \left[-\frac{1}{2k_z} \nabla_{\perp}^2 + \frac{1}{2k_z} (k_z^2 - k_0^2 n^2(x, y)) \right] \varphi(x, y, z) \quad (2)$$

The Helmholtz equation satisfying the light field in the continuum is extended to the waveguide model to obtain the coupled mode equation

$$i \frac{\partial}{\partial z} \varphi_n(z) = \sum_m c_{nm} \varphi_m(z) \exp[-j(\beta_n - \beta_m)z] \quad (3)$$

Equation (3) describes evolution equations for the optical field φ_n in each waveguide. c_{nm} denotes the coupling coefficients between the n -th and the m -th waveguides, $\beta_{n/m}$ is the propagation constant of the n -th / m -th waveguide. When the tight-binding approximation is performed on the coupled mode Equation (3), the coupled mode equation describing the waveguide array can be obtained by

$$-i \frac{\partial}{\partial z} \varphi_n(z) = \beta_n \varphi_n(z) + c_{n,n-1} \varphi_{n-1}(z) + c_{n,n+1} \varphi_{n+1}(z) \quad (4)$$

Equation (4) is also formally equivalent to the Schrödinger equation under tight-binding approximation. Therefore, the physical phenomena described by the Schrödinger equation can be simulated by waveguide arrays. Almost all tight-binding Hamiltonian systems can be implemented by controlling the waveguide refractive index and waveguide spacing. Note that, the propagation direction z plays the role of the temporal coordinate, so the dynamics of electromagnetic waves can be experimentally observed by the waveguide arrays.^[39] The analogs of quantum Hall TIs requiring breaking the TR symmetry may be realized by spatially modulating waveguide shape along the z direction.

2.2. Fabrication and Optical Measurement of Waveguide Arrays

There are three types of waveguides arrays used to perform topological photonic experiments: waveguide arrays in transparent solid materials (e.g., glass and crystal), silicon waveguide arrays, and plasmonic waveguide arrays, as shown in **Figure 1**. In the following, we summarize the fabrications, measurements, advantages, and disadvantages of these waveguide arrays, so that readers can find the suitable one.

Waveguide arrays in transparent materials are generally fabricated using femtosecond laser direct writing (FSDW) technology. FSDW is an efficient and convenient technology that uses multiphoton absorption to induce the modification of the laser-irradiation region in transparent materials, enabling the fabrication of waveguides.^[62] The whole preparation process is performed by a femtosecond laser beam, a computer-controlled 3D xyz translation stage, and a microscope objective. To write the waveguide array, the fs beam is focused inside the transparent material with a microscope objective, and the transparent material is placed on a 3D xyz translation stage. Some beam shaping techniques (e.g., slit beam shaping, astigmatic beam shaping, and spatial light modulator beam shaping) are essential for shaping the circular cross-section of the waveguide. The waveguide refractive index is controlled by the fs laser pulse energy and the moving speed of the translation stage. The waveguide array structures can be changed freely by altering the position of the laser focus. Benefiting from the flexible fabrication technique, most topological models can be realized by waveguide arrays written in glass, including the chain lattice model,^[63,64] Floquet lattice

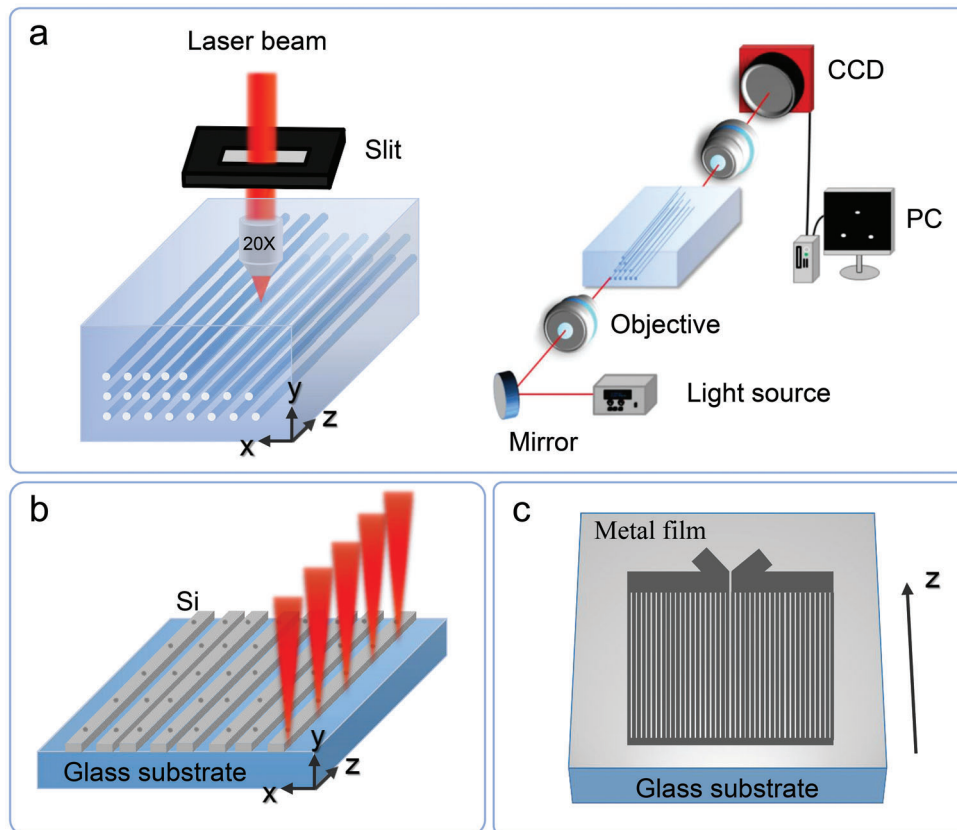


Figure 1. Schematic showing the fabrications and measurements of the a) laser direct writing waveguide arrays, b) silicon waveguide arrays, and c) plasmonic waveguide arrays. (a) Reproduced with permission.^[80] Copyright 2019, Springer Nature.

model,^[39,65,66] honeycomb lattice model,^[67,68] and square lattice model.^[69,70] After completing the writing process, a light beam is injected into the lattice, and the diffraction behavior of the light in the waveguide array is measured by a microscope objective and a charge-coupled device (CCD) camera (Figure 1a).

Writing waveguide arrays in transparent materials is not limited to fs lasers. Chen fabricated a series of topological waveguide arrays in crystal using the continuous-wave (CW) laser writing technique.^[71–76] These waveguide lattices can be more readily reconfigurable in terms of lattice spacing and boundary structures, compared with the (FSDW) method. Some novel topological phenomena, such as flatband line states,^[71] noncontractible loop states,^[74] and nonlinear topological valley Hall edge states,^[75] have been observed with the CW direct-writing waveguide arrays.

Silicon waveguide arrays are fabricated using the method of E-beam lithography and the inductively coupled plasma etching process. The fabricating process consists of the following steps: (i) depositing a certain thickness of silicon on a glass substrate, then cleaning and drying it; (ii) spin-coating the photoresist film onto the substrate; (iii) the substrate is exposed to the electron beam in E-beam writer to form the nanostructures; (iv) transferring the substrate into the inductively coupled plasma etcher for waveguide array etching. For the optical measurements in silicon waveguide arrays, the input grating couplers are indispensable for inputting the light signals. The output light can be detected

by the scattering field from the hole patterns formed in the waveguides (Figure 1b). The silicon waveguide arrays have been used to explore the topological boundary states in the Hermitian and non-Hermitian lattices.^[77–79] But limited by 2D planar lithography, it only applies to 1D topological systems.

The plasmonic waveguide arrays based on the coupling of surface plasmonic polaritons (SPPs) have a similar fabrication method to silicon waveguide arrays, except that the substrate is coated with a metal film. Compared with the dielectric ones, the plasmonic waveguide arrays offer a much wider range of couplings but more enormous propagation losses. Optical measurements require converting input spatial light into surface plasmons through a coupled grating, and the output light in the waveguide array is observed through slit scattering at the array end (Figure 1c). The tiny size of the plasmonic waveguide arrays is very beneficial for incorporating topological phenomena into photonic devices.

In comparison, the laser-direct-writing waveguide arrays are flexible in fabrication and can be used to construct complex topological models. However, the slight refractive index contrast between the waveguide and the matrix material results in a larger sample size, making it challenging to achieve high-density integration. Therefore, the waveguide arrays in transparent materials are an appropriate platform for simulating topological photonic phenomena. Silicon waveguide arrays and plasmonic waveguide

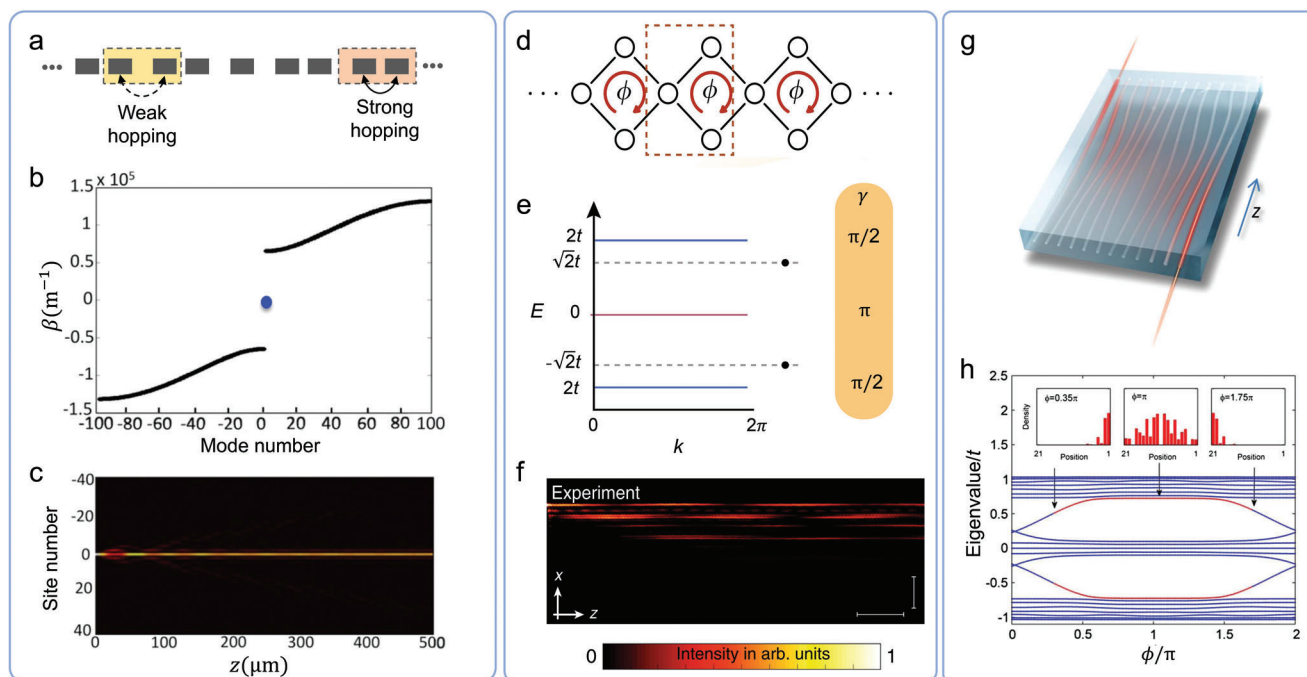


Figure 2. Light dynamics in the 1D topological waveguide arrays. a) Schematic showing the connection of two Su–Schrieffer–Heeger (SSH) models with distinct topological phases. b) The eigenspectra of the finite waveguide array with the structure in (a). The zero mode lies in the bulk band gap. c) Light propagation in the structure of (a) when the zero-mode waveguide is excited. d) Schematic showing a 1D square-root topological insulator (TI). e) The energy dispersion of the finite chain with the structure in (d). Two in-gap boundary states appear at $E = \pm\sqrt{2}t$, where t donates the hopping strength between the two nearest neighbor sites. f) Boundary dynamics in the 1D square-root topological waveguide arrays. g) An illustration of the adiabatically modulated photonic quasicrystal. h) The spectrum of the quasicrystal model as a function of the modulation phase ϕ . (b,c) Reproduced with permission.^[77] Copyright 2016, American Physical Society. (d-f) Reproduced with permission.^[85] Copyright 2020, Springer Nature. (g,h) Reproduced with permission.^[86] Copyright 2016, American Physical Society.

arrays, which are highly integrated but limited by 2D manufacturing technology, are more suitable for fabricating robust optoelectronic integrated devices.

3. Topological Phases in the Photonic Waveguide Arrays

Since the semi-infinite photonic superlattice was first introduced to demonstrate the Shockley-like surface states in the Su–Schrieffer–Heeger (SSH) model,^[72] the curtain of experimentally studying the topological phases by using the waveguide arrays has been opened. Complex topological phenomena, such as edge states for unidirectional transport, localized boundary states, and topological vortex zero modes, have been observed in the arrays. We classify the topological states realized in the waveguide arrays according to their system dimensions, and review them as follows.

3.1. One-Dimension Topological System

In 1D TIs where the existence of topological phases depends on the symmetries of the system, the prototypical one is the SSH model.^[81] The model is a dimeric chain with alternating strong and weak hopping strengths, exhibiting a topologically

nontrivial phase for weak intracellular hopping and a topologically trivial phase for strong intracellular hopping. The different topological phases can be distinguished by the Zak phase Z (or winding number W). The non-zero Zak phase in the finite chain implies the existence of the topological boundary states. For the SSH waveguide arrays, the topological phase transition depends on the modulation of the waveguide spacing. As shown in **Figure 2a–c**, when two SSH waveguide arrays with distinct topological phases are connected, a topologically protected zero mode appears in the band gap. The light injected into the zero-mode waveguide remains localized.^[77,82–84] Subsequently, the SSH waveguide arrays have been combined with quantum optics to achieve the measurements of topological invariants of optical systems. Wang et al. directly observed topological phase transition points by using bulk-state photon dynamics.^[63] The winding number $W = 1$ for the topological nontrivial system and $W = 0$ for the topological trivial system are obtained by integrating several waveguide lattices with different evolution lengths on a chip. The primary attempt, combining topology with quantum-integrated photonics, provides a reference scheme for realizing scalable topologically protected quantum information processing.

Beyond the standard SSH model, other intriguing lattices have been implemented in the 1D waveguide arrays, such as the multi-band photonic superlattice,^[64,78] photonic Aubry–André lattice with two-photon quantum correlation states,^[87,88] gauge-shifted Floquet photonic lattice,^[89,90] and quasi-periodical lattice with

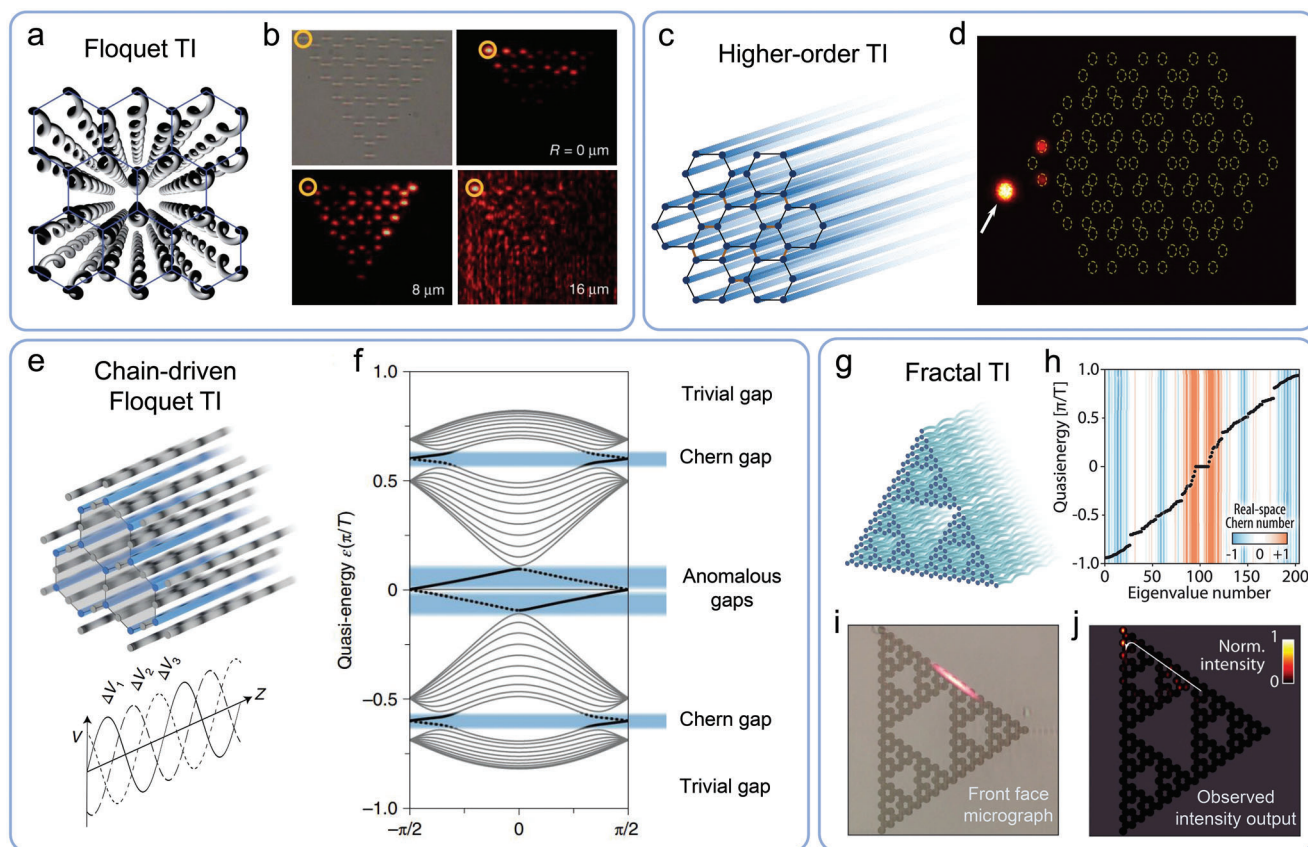


Figure 3. The topological phases in the 2D waveguide arrays. a,b) Geometry and experimental results of honeycomb photonic Floquet topological waveguide arrays. The light diffractions in the structures with various helix radius R are shown in (b), the yellow circles indicate the position of light injection. c) Geometry of the honeycomb higher-order topological waveguide arrays, and d) light diffractions in the structure when the zero-mode is excited. In (d), dashed circles mark the positions of the waveguides. e) The band structures of the chain-driven Floquet TIs. g) The Floquet fractal TI and h) its energy band. i,j) Topological edge transport in the Floquet fractal waveguide arrays. The unidirectional counterclockwise propagation is observed. (a,b) Reproduced with permission.^[39] Copyright 2013, Springer Nature. (d) Reproduced with permission.^[103] Copyright 2018, Macmillan Publishers (e,f) Reproduced with permission.^[105] Copyright 2022, Springer Nature. (g–j) Reproduced with permission.^[106] Copyright 2022, American Association for the Advancement of Science.

topological Anderson phase.^[91] In particular, Kremer presented a novel square-root TI implemented in the waveguide arrays with a specifically tailored effective negative hopping.^[85] The square-root TIs exhibit robust boundary states, but their bulk bands have a nonquantized topological index (Zak phase $Z = -\pi/2$), as shown in Figure 2d–f. This surprising finding is different from the conventional robust boundary effects.

Another fascinating structure implemented in waveguide arrays is the 1D quasicrystal, which can be described as a projection of a 2D lattice on a line. In 2012, Kraus first built 1D Harper quasicrystals and realized Thouless pumps using the waveguide arrays.^[86] In their implementation, the waveguide spacing was altered along the propagation direction, giving rise to the modification in the modulation phase ϕ of the Harper model (Figure 2g,h). A set of samples varying in length with equal spacing was fabricated to experimentally scan ϕ . The light intensity distributions at the output surface of all samples reconstruct the light trajectory along the full adiabatic process. Subsequently, Verbin extended the topological pumps to the Fibonacci chain, and observed the phase transitions between two topologically distinct quasicrystals using light propagation in the waveguide

array.^[92] Note that, although these quasicrystals are implemented in 1D waveguide systems, their topological boundary states are described by the 2D topological invariants (known as Chern numbers). This is due to the fact that the topological bulk response of the 1D pump in quasicrystals matches that of the 2D quantum Hall effect. The study of quasicrystals can give rise to further experimental realizations of topological phases in higher dimensions.^[93,94]

3.2. Two-Dimension Topological System

The experimental demonstration of the 2D TIs in the waveguide arrays was first proposed by Rechtsman.^[39] They implemented a Floquet TI using the helical waveguides array, where the helicity of the waveguides breaks z -reversal symmetry (Figure 3a). The topologically nontrivial states can be induced via “ z -periodic” driving, rather than via magnetic or spin–orbit effects. Thus, a topologically protected chiral edge state can be observed by tuning the helicity radius, as shown in Figure 3b. Similar helical waveguide arrays have been used to exhibit complete collapse of

quasi-energy bands,^[67] topologically protected transport induced by adding onsite disorder,^[95] and photonic topological states in synthetic space.^[96] In general terms, the periodic driving is weak, as increasing the bend degree results in more propagation losses in the waveguide. The topological properties of the Floquet TIs are still described by Chern number, as in the static system. To explore topological phenomena that have no counterpart in the static Hamiltonian, an anomalous Floquet TI was proposed, which hosts chiral edge states despite the vanishing Chern number. The idea is implemented in the helical waveguide arrays with staggered lattices, where the topological phase transitions can be tuned by varying the waveguide spacing rather than the bending radii.^[40,97] The anomalous Floquet topological phases exhibit rich topological physics and unusual features, e.g., robust anomalous chiral edge states and perfectly localized bulk states. The long-time dynamics of these expanded and localized states were demonstrated by a state-recycling technique.^[98] The same design also contributed to observing “type-II” Weyl points of light and Fermi arc-like surface states.^[99] Especially, the *z*-modulated waveguide arrays can realize a 2D topological pump,^[100] arousing the topological characteristics of the 4D quantum Hall system.

Besides the Floquet TIs, Rechtsman et al. demonstrated graphene-like TIs and valley Hall photonic TIs using straight waveguide arrays.^[49,68] The zigzag edge states can be observed in both structures by detuning the spaces or refractive indices of the two honeycomb sublattices. The edge states support counter propagation and are robust only against certain classes of disorder due to the invariance of the RT symmetry. Moreover, the stationary waveguide arrays were used to demonstrate a topological mode bound to a vortex, giving rise to direct observation of the characteristic topological vortex zero mode and geometric phase.^[101,102]

As an extension of TIs, higher-order topological phases, obeying unconventional bulk-boundary correspondences, were proposed. Compared with TIs, the higher-order topological insulator (HOTI) exhibits unusual topological characteristics. For instance, a 2D HOTI hosts 0D corner states. The first experimental implementation of photonic HOTI was achieved in the honeycomb waveguide arrays by Noh.^[103] The structure is constructed as a hexagon with six corners, as shown in Figure 3c. An auxiliary waveguide with zero mode is used to confirm the existence of 0D corner-localized mid-gap modes (Figure 3d). Similar higher-order topological corner states were later realized in the breathing kagome waveguide array,^[80] square waveguide array,^[69,70] decorated honeycomb waveguide array,^[73] and helical waveguide array.^[104] This class of corner states is generally topologically protected by spatial symmetries.

Most recently, the topological phase structures with richer physics have been implemented in waveguide arrays. One is bi-morphic Floquet TIs in the chain-driven honeycomb lattice.^[105] Unlike conventional Floquet TIs that require bending or stretching of optical waveguides, Floquet driving of the chain-driven TIs is achieved by introducing interstitial elements between the sites of a static periodic lattice (Figure 3e). By periodically modulating onsite potentials of the interstitial elements, multiple topological phases, including the conventional Chern topological phase and anomalous topological phase, can exist simultaneously in a structure (Figure 3f). Another is fractal TIs composed exclusively of edge sites using helical waveguide arrays.^[106] The TIs host topo-

logically protected chiral edge states when driven by Floquet, even without any bulk structure (Figure 3g–i). The design of fractal TIs goes beyond the concept of bulk-boundary correspondence and is expected to inspire the synthesis of advanced topological materials.

4. Non-Hermitian Effect in the Topological Waveguide Arrays

The topological models introduced in the above section are based on Hermitian systems. At the same time, the non-Hermitian systems with open boundaries are also worth studying due to their closer resemblance to the real systems. Under appropriate conditions, non-Hermitian systems interact with topological physics and contribute unique topological properties. Generally, the topological phases in the non-Hermitian systems are closely related to parity-time (PT) symmetry. A PT-symmetry system, which obeys the relation $[PT, H] = 0$, can possess real eigenvalues in the presence of gains and losses. An eigenvalue transformation from real to complex numbers signs the occurrence of the PT symmetry phase transition. The transition point is called the exceptional point (EP).

In parallel to the progress on topological states in the conventional photonic waveguide systems, there has been plenty of recent research into non-Hermitian systems. Most experimental implementations in 1D non-Hermitian waveguide arrays are based on non-Hermitian SSH model, as shown in Figure 4a. Following the coupled mode Equation (3), the optical field propagation within the non-Hermitian waveguide lattice can be described by

$$\begin{aligned} -i \frac{\partial}{\partial z} \varphi_n &= \beta_l \varphi_n(z) + c_2 \varphi_{n-1} + c_1 \varphi_{n+1}, \quad n = 1, 3, 5, \dots \\ -i \frac{\partial}{\partial z} \varphi_n &= \beta_g \varphi_n(z) + c_1 \varphi_{n-1} + c_2 \varphi_{n+1}, \quad n = 1, 3, 5, \dots \end{aligned} \quad (5)$$

where propagation constants in the lossy and gain waveguides $\beta_l = \beta_0 + i\gamma$, $\beta_g = \beta_0 - i\gamma$, and γ is the loss or gain strength. c_1 and c_2 are two coupling coefficients of the nearest neighbor waveguide. The corresponding tight-binding Hamiltonian can be expressed as

$$H = \begin{pmatrix} i\gamma & c_1 + c_2 e^{ika} \\ c_1 + c_2 e^{-ika} & -i\gamma \end{pmatrix} \quad (6)$$

k is the wavenumber, and a is the period. Eigenvalues β are solved as

$$\beta^2 = c_1^2 + c_2^2 + 2c_1c_2 \cos(ka) - \gamma^2 \quad (7)$$

Here, if $|c_1 - c_2| > \gamma$, all eigenvalues are real. Otherwise, the EP appears at the position $\cos(ka) = \frac{\gamma^2 - c_1^2 - c_2^2}{2c_1c_2}$ (Figure 4a). The theory provides a reference for preparing photonic lattices with PT-symmetry broken and unbroken phases.

The first experimental observation of the topological transition in the non-Hermitian system was presented by Zeuner.^[107] They defined a relationship between the phase transition and the mean displacement of light diffraction in the lattice. Later, the topological defect mode in the PT-symmetric SSH waveguide arrays

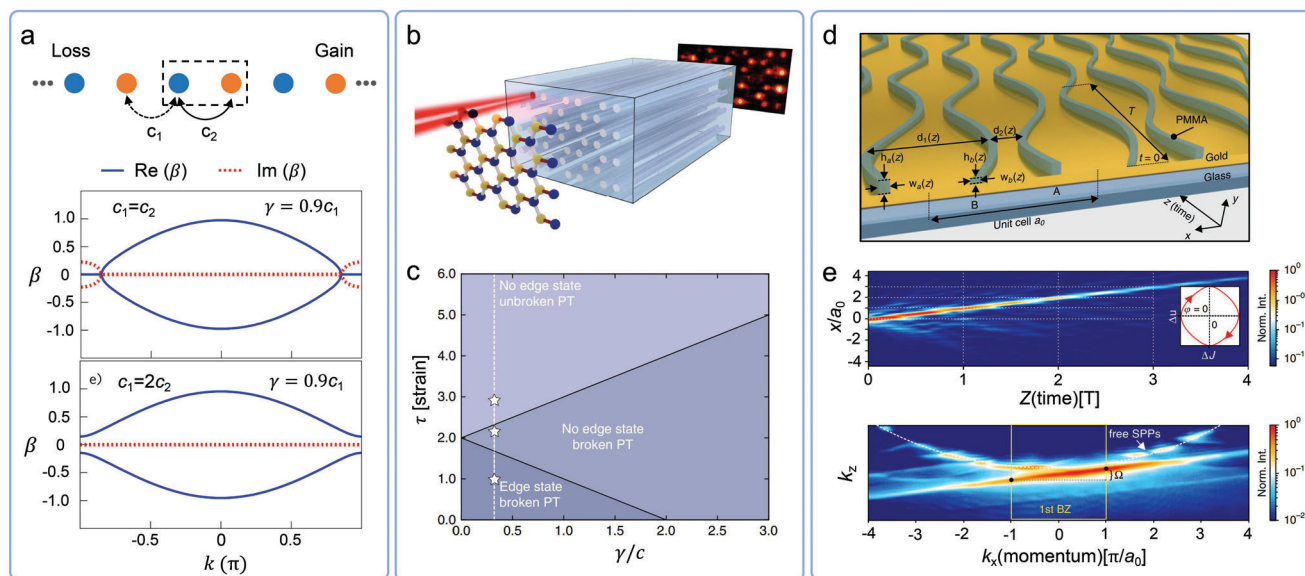


Figure 4. Non-Hermitian topological waveguide systems. a) The non-Hermitian Su–Schrieffer–Heeger (SSH) lattice and its dispersion. b) The parity-time (PT) symmetric graphene lattice. c) The relation diagram of the PT-symmetry phase transition and the topological phase transition, τ represents a linear strain applied on a coupling term (red connections in (b)). d) The non-Hermitian Rice–Mele waveguide lattice. e) Observation of fast Thouless pumping in the array of (d). For all z , we can observe a strongly localized wave packet, whose center of mass is transported in a quantized manner. (a) Reproduced with permission.^[108] Copyright 2017, Macmillan Publishers. (b,c) Reproduced with permission.^[112] Copyright 2019, Macmillan Publishers. (d,e) Reproduced with permission.^[114] Copyright 2020, Springer Nature.

was predicted and observed.^[108–111] The defect mode is localized at the interface between two topologically distinct PT-symmetric photonic lattices. Unlike the Hermitian systems where the zero modes may deviate from the exact-zero energy in finite systems, the defect modes in the non-Hermitian systems always remain motionless at the zero energy due to the enhanced protection of PT-symmetry phase transition. Moreover, the non-Hermitian-enhanced topological protection is proven to recover the deviated zero-mode of finite Hermitian lattices.^[79] The non-Hermitian waveguide arrays can be expanded to 2D by judiciously designing the microscopic scattering points as the losses, providing a more versatile platform for exploring the non-Hermitian band topology. For instance, a 2D PT-symmetry graphene lattice demonstrates the relation between the PT-symmetry phase transition and the topological phase transition^[112] (Figure 4b,c); a PT-symmetric Lieb lattice with ribbon shape and complex couplings demonstrates the higher-order EP and Landau–Zener Bloch oscillations.^[113]

As mentioned earlier, breaking the z -reversal symmetry of waveguide arrays is another way to endow physical systems with topologically nontrivial phases. When generalizing the Floquet topological theory to non-Hermitian systems, the interplay of Floquet topological phases and non-Hermiticity leads to more exotic effects that have no counterpart in static or Hermitian systems. One example is topological transport quantization in fast Thouless pumps.^[114] For the problem that fast driving of the Hermitian system always destroys the quantization of Thouless pumping, time-periodic and space-periodic losses were found to be capable of leading to restoration of quantized transport (Figure 4d,e). In addition, a related periodic loss can open a topo-

logically nontrivial gap and create a Floquet π mode.^[115] Non-Hermiticity and Floquet engineering provide higher degrees of freedom for topological photonic systems, inspiring more potential applications.

Despite this progress, some challenges remain for the non-Hermitian experimental implementation in waveguide arrays. The first is how to build a system with real gains and losses. While losses can be precisely regulated, there is currently a lack of methods to introduce and control real gains. Second, how to define and measure topological invariants for the non-Hermitian Hamiltonians. A leaky photonic lattice has been proposed to measure the bulk topological invariants related to the Hermitian system, but whether it can be applied to non-Hermitian systems has not been verified.^[116] These questions should be considered in future research.

5. Nonlinear Effect in the Topological Waveguide Arrays

Thus far, most of the research in topological photonics has focused on linear topological effects, whereas the introduction of nonlinear effects may bring other novel physical phenomena. In topological photonics, nonlinear waveguide arrays are characterized by a focusing Kerr response. One of the advantages of optical waveguides is that the dynamics are governed by the ratio of the nonlinearity to the linear coupling coefficient. In other words, the effective nonlinearity can be increased by reducing the coupling of the waveguides with the long propagation distance. However, after the theoretical prediction,^[117] it was almost a decade until the nonlinear localized states were experimentally observed in

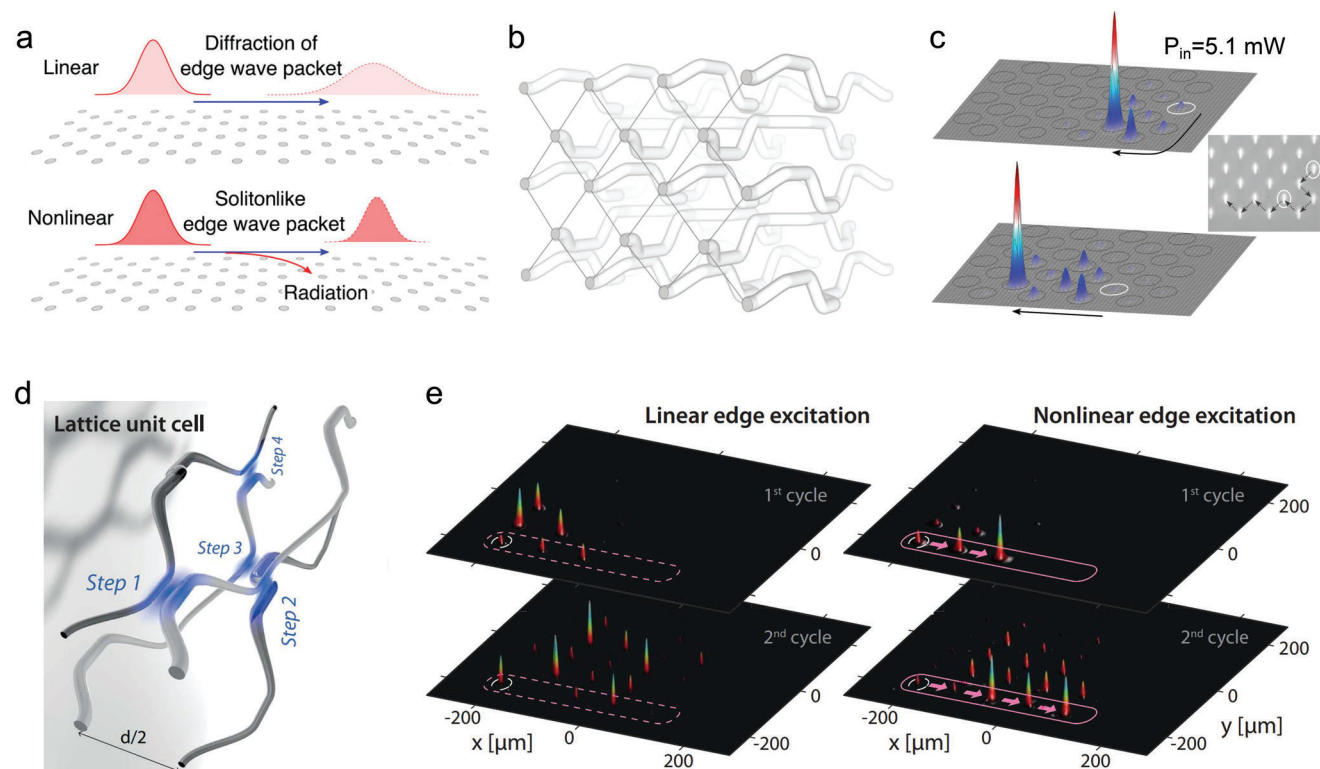


Figure 5. Edge solitons and nonlinearity-induced topological edge states. a) Schematic showing the edge wave packet in the linear and nonlinear regime. b) The geometry of anomalous Floquet square topological lattice. c) Soliton edge states unidirectionally propagating around a corner under a high nonlinearity. White circles indicate the position of the light injection. d) Anomalous Floquet waveguide array with designed waveguide diameters along the propagation direction. e) Light diffractions in the anomalous Floquet arrays under the linear and nonlinear edge excitation regime. Unidirectional edge transport is observed at the nonlinear excitation. (a–c) Reproduced with permission.^[124] Copyright 2021, American Physical Society. (d,e) Reproduced with permission.^[121] Copyright 2020, American Association for the Advancement of Science.

the waveguide arrays. In the nonlinear waveguide arrays, the optical field propagation can be described by

$$-i\frac{\partial}{\partial z}\varphi_n(z) = \sum_{(m)} H_{m,n}(z) \varphi_m(z) + \chi^2 |\varphi_n(z)|^2 \varphi_n(z) \quad (8)$$

where $H_{m,n}(z)$ is the linear tight-binding Hamiltonian, describing the coupling from the n -th to the nearest neighbor m -th waveguide. χ^2 denotes the strength of the Kerr-type nonlinearity, and $|\varphi_n(z)|^2$ is the optical power at the n -th waveguide. Evidently, for low input powers, the nonlinear term $\chi^2 |\varphi_n(z)|^2 \varphi_n(z)$ is negligible. With increasing input power, the nonlinear term becomes vital for the dynamic of light in topological waveguide arrays and may trigger some novel phenomena, such as nonlinear topological solitons,^[118–120] nonlinearity-induced topological phase transitions,^[121] and nonlinearity-controlled switching.^[122]

The experimental research on nonlinearity in the waveguide array mainly focused on two systems. One is the anomalous Floquet TIs. Lumer reported the first theoretical prediction in the nonlinear Floquet topological photonics in 2013, which highlighted a long-lived and self-localized soliton residing in the bulk of the system.^[123] Such a soliton continuously rotates with a predetermined directionality around the edge of the defect. Due to technical limitations, only recently, the cyclotron-like soliton was experimentally observed in the nonlinear waveguide arrays.^[118,124]

These solitons can unidirectionally propagate around a corner of the lattice without backscattering (Figure 5a–c). More interestingly, nonlinearity can drive the topological phase transition from an initially topologically trivial phase to a transient topological phase, inducing a nonlinear chiral edge state^[121] (Figure 5d,e). Despite the transient nature of the nonlinearity-induced topological phase, the edge states are robust and persist even in the presence of disorder and defects.

Another focus is higher-order topological systems. In the 2D higher-order TIs, corner states can tightly localize light. Then the question arises whether such localized states can contribute to enhancing nonlinear effects. For this question, nonlinear higher-order topological corner states in the kagome and square lattice were studied.^[125,126] The authors fabricated three types of waveguide arrays with topologically nontrivial phases, homogeneous, and topologically trivial phases. For topologically nontrivial phases, with strong nonlinearity, the corner states become more localized and form solitons in both kagome and square lattices. However, for the homogeneous and topologically trivial cases, the excited light in the kagome lattice remains confined to the initially injected waveguides, while the excited light in the square lattice diffracts into the edge and bulk. The different behavior may be related to the different band properties of the two models. Indeed, more experimental explorations are needed to help understand the nonlinear higher-order topological phases.

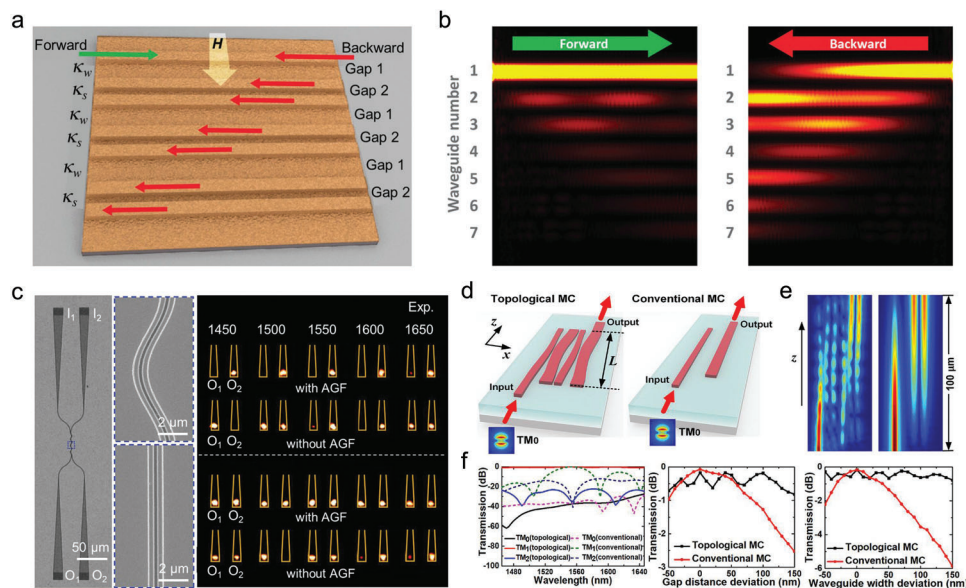


Figure 6. Applications of topological waveguide arrays. a) Schematic of the magneto-optic isolator based on the Su-Schrieffer-Heeger (SSH) waveguide arrays. b) Topological edge-state propagation in the magneto-optic isolator for forward and backward directions. c) The optical coupler realized by artificial gauge field (AGF) based silicon waveguide platform, the broadband, and robust routing functions was verified. d) Schematic showing the structures of topological mode converter (MC) based on the Thouless pumping and conventional MC. e) Propagation profiles for the topological MC (left) and conventional MC (right). f) Simulated transmission spectra of different modes at the output ports of the topological and conventional MCs when varying the wavelength, waveguide gap distance, and waveguide width. (a,b) Reproduced with permission.^[128] Copyright 2019, American Physical Society. (c) Reproduced with permission.^[132] Copyright 2022, American Physical Society. (d-f) Reproduced with permission.^[133] Copyright 2022, Wiley-VCH GmbH.

There have also been some studies of nonlinearity beyond Floquet TIs and higher-order TIs. For instance, the nonlinearity of the non-Hermitian topological systems based on the SSH model can induce the transition between PT-symmetric and PT-symmetric broken regimes, realizing a topological zero mode;^[127] a type-II Dirac photonic lattice hosts topologically protected nonlinear valley Hall edge states;^[75] edge solitons in the trimer waveguide arrays can be driven into the upper allowed band under nonlinearity.^[120] These experimental studies on nonlinear topological effects provide a reference for condensed matter physics and topological materials, and bring more possibilities for developing photonic technologies.

6. Topological Waveguide Arrays Toward Applications

The novel topological effects of waveguide arrays open a new route for the implementation of topological photonics in practical applications. The most straightforward applications involve topologically protected boundary states as high-performing photonic integration devices.^[128–130] As shown in **Figure 6a,b**, the topological SSH waveguide arrays fabricated on magneto-optical materials can serve as optical isolators with much higher isolation ratios than any other on-chip isolator device types, due to the intrinsic nonreciprocity of the magneto-optic effect.^[128] The same structure fabricated on a silicon substrate can be developed into a broadband optical coupler.^[130] Besides static waveguides, topological coupled-waveguide devices induced by artificial gauge field (AGF) and Thouless pumping mechanism exhibit more advantages, in terms of robustness against the variations in wavelength, waveguide spacing, and waveguide width

(Figure 6c).^[131–133] These predominant properties arise from the topological protection of the Chern number and the flat dispersion. In particular, the driven topological waveguide arrays can be extended to build the mode converters (MCs).^[133] These devices are remarkably robust against structural fluctuations and have a broad working bandwidth compared to their conventional counterparts, suggesting application potentials in broadband-integrated photonics (Figure 6d–f). Whereas the adiabatic requirement of Thouless pumps leads to a large footprint, the robustness of the topological device contradicts the compactness.

The introduction of the non-Hermitian systems would presumably resolve this contradiction. Some works have revealed that dynamically encircling EPs by adiabatically varying the parameters could lead to asymmetric mode transfer in a lossy waveguides system, thus realizing asymmetric mode/polarization converters.^[134–138] Although the adiabaticity condition is still required for this system, the adiabatic approximation can be achieved via a fast parametric evolution along the boundary of the non-Hermitian Hamiltonian parameter space. Therefore, the mode-switching devices, requiring a small footprint, can be developed in the non-Hermitian regime. Such dynamics in encircling EPs are expected to extend to non-Hermitian waveguide arrays with more complex and interesting topology,^[139] holding great promise for developing wideband MCs with both a small footprint and robustness against the structure perturbation.

7. Conclusions and Outlook

Research activities on topological photonics have grown remarkably in recent years, and evanescently coupled optical waveguide

arrays have made outstanding contributions to their experimental implementation. Here, we present an overview of how optical waveguide arrays provide a versatile experimental platform to implement various topological systems, including 1D TIs, quasicrystals, Floquet TIs, valley Hall TIs, and higher-order TIs. Unlike other topological photonic platforms (e.g., photonic crystals slab and coupled resonant ring), the waveguide arrays can provide the direct observation of light dynamics due to the equivalence between the propagation direction of the waveguides and the time coordinate. We particularly discussed novel physical phenomena arising from the combination of topology with non-Hermitian and nonlinearity (e.g., breakup and recovery of topological zero modes, topological transport quantization, optical soliton, and nonlinearity-induced transient topological phases), which we believe are significant for the comprehensive understanding of topological physics and practical applications.

In the coming years, we expect the topological waveguide arrays to find more applications in topological photonic devices. The topological waveguide arrays have the ability to manipulate light and are immune to defects and disorder, providing a promising solution for fabricating robust photonic integrations devices. However, the current applications of topological waveguide arrays are mainly focused on passive devices. Only theoretical reports exist for active photonic devices such as topological waveguide lasers.^[140–142] The factors that hinder active topological waveguide devices are as follows: (i) introducing gain into a waveguide is challenging due to its elongated shape; (ii) the long propagation distances of light in the waveguides result in enormous losses, making it difficult to achieve positive gain even if the gain is successfully introduced. We envision that nonlinear modulation in topological states may provide an efficient solution to these problems.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

coupled waveguide arrays, non-Hermitian system, nonlinear effects, topological photonics, topologically nontrivial phases

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