

Observation of \mathcal{PT} -Symmetry Breaking in Complex Optical Potentials

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In 1998, Bender and Boettcher found that a wide class of Hamiltonians, even though non-Hermitian, can still exhibit entirely real spectra provided that they obey parity-time requirements or \mathcal{PT} symmetry. Here we demonstrate experimentally passive \mathcal{PT} -symmetry breaking within the realm of optics. This phase transition leads to a loss induced optical transparency in specially designed pseudo-Hermitian guiding potentials.

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In the Dirac–von Neumann formulation of quantum mechanics, all physical observables must be represented by self-adjoint or Hermitian operators on a Hilbert space [1]. The Hamiltonian of a system is of course no exception. This last condition not only leads to real energy eigenvalues but also guarantees that the wave function norm remains invariant with time. Interestingly, a decade ago, Bender and colleagues have theoretically shown that a wide class of non-Hermitian Hamiltonians, the so-called \mathcal{PT} -symmetric Hamiltonians, can also have an entirely real spectrum [2]. The parity operator \hat{P} , responsible for spatial reflections, is defined through the operations $\hat{p} \rightarrow -\hat{p}$, $\hat{x} \rightarrow -\hat{x}$, while the time reversal operator \hat{T} leads to $\hat{p} \rightarrow -\hat{p}$, $\hat{x} \rightarrow \hat{x}$ and to complex conjugation $i \rightarrow -i$. In general, we state that the \mathcal{PT} symmetry of a system is unbroken provided that the eigenfunctions of a complex Hamiltonian are also eigenfunctions of the \mathcal{PT} operator. In this case, it can be directly shown that a necessary condition (but not sufficient) for a Hamiltonian to be \mathcal{PT} -symmetric is that the potential associated with it obeys $V(x) = V^*(-x)$. More specifically, \mathcal{PT} reflection requires that the potential energy operator is even in its real part while odd in its imaginary. One of the most interesting effects associated with this class of Hamiltonians is the onset of a phase transition behavior arising from a spontaneous breakdown of \mathcal{PT} symmetry, beyond which the spectrum becomes complex [2–4]. Pseudo-Hermitian \mathcal{PT} symmetry has led to new developments in diverse areas of theoretical physics, including quantum field theories [3], Lie algebras [5], and complex crystals [6,7]. Even though wave propagation in complex potentials has been studied before [8–11], the emergence

of a \mathcal{PT} -phase transition has never been considered. In this Letter, we provide the first demonstration of a passive \mathcal{PT} -symmetry breaking in optical complex potentials.

Quite recently, the prospect of realizing complex \mathcal{PT} -symmetric potentials within the framework of optics has been suggested [12–14]. What makes this possible is the formal equivalence between the quantum mechanical Schrödinger equation and the optical wave equation. To appreciate this fact, let us consider a 1D planar inhomogeneous configuration with a complex refractive index distribution $n_0 + n_R(x) + in_I(x)$, where n_0 represents a constant background index, $n_R(x)$ is the real index profile of the structure, and $n_I(x)$ stands for the gain or loss component. Note that for the systems discussed here $n_0 \gg n_{R,I}$. In this case, the electric field $E = \Psi(x, z) \exp[i(\omega t - kz)]$ of a light wave propagating in this planar weakly guiding arrangement satisfies a Schrödinger-like equation:

$$i \frac{\partial \Psi}{\partial z} = \hat{H} \Psi, \quad (1a)$$

where the optical Hamiltonian is given by

$$\hat{H} = \frac{1}{2k} \frac{\partial^2}{\partial x^2} + V(x), \quad (1b)$$

where $k = k_0 n_0$, $k_0 = 2\pi/\lambda_0$, with λ_0 being the vacuum wavelength of light. On the other hand, the optical potential appearing in Eq. (1b) is given by $V(x) = k_0[n_R(x) + in_I(x)]$. Evidently, the Hamiltonian of Eq. (1b) is \mathcal{PT} -symmetric provided that the real index part is an even function of position $n_R(x) = n_R(-x)$ while the imaginary is odd, i.e., $n_I(x) = -n_I(-x)$, [12]. Thus complex optical \mathcal{PT} potentials can be realized by judiciously in-

tegrating an antisymmetric gain and loss profile on an even index distribution [12]. As in quantum mechanics, the optical eigenmodes $\Psi = \Phi(x) \exp(-i\epsilon z)$ associated with these potentials can be obtained from the stationary Schrödinger problem $\hat{H}\Phi = \epsilon\Phi$, where the mode propagation constant β is given by $\beta = k + \epsilon$.

As previously indicated, the condition $V(x) = V^*(-x)$ is necessary for \mathcal{PT} reflection but not sufficient. This should be augmented with a requirement of common eigenfunctions for both \hat{H} and $\hat{P}\hat{T}$. As the \mathcal{PT} operator is not linear, commutation between the Hamiltonian and the \mathcal{PT} operator does not guarantee a real spectrum [15]. Violation of this property is known as spontaneous \mathcal{PT} -symmetry breaking. Broken \mathcal{PT} symmetry typically involves the unfolding of an eigenfunction into complementary eigenfunctions at the so-called *exceptional point* [16], previously studied in several fields of physics [17,18]. After this point, the spectrum ceases being entirely real and becomes complex, which marks the onset of a phase transition. To illustrate this effect, let us consider a \mathcal{PT} -symmetric ridge optical waveguide as shown in Fig. 1. For demonstration purposes, let the core ($n \cong 3.28$) and the cladding ($n \cong 3.25$) substrate of this composite structure be AlGaAs. This symmetric guiding element is operated at a wavelength of $\lambda_0 = 1.55 \mu\text{m}$ with one-half of it experiencing gain while the other half an equal amount of optical loss. The effective index contrast of this element is approximately $\Delta n = 4.6 \times 10^{-4}$ (weakly guided), and the level of gain or loss is varied between $\pm(0-70) \text{ cm}^{-1}$. Given the imposed complex refractive index distribution, this optical potential satisfies the aforementioned \mathcal{PT} -symmetry requirements. Figure 1(a) depicts the modal intensity distribution below the phase transition point α_c —the critical optical loss coefficient. In this range, the mode intensity is symmetric with respect to the mirror axis, and the spectrum of this system (including radiation modes) is real. This \mathcal{PT} symmetry is spontaneously broken once the gain/loss contrast exceeds $\alpha_c \cong 50 \text{ cm}^{-1}$, and hence the set of eigenvalues becomes partly complex. Above this critical point, this fundamental mode bifurcates into two asymmetric modes, one residing in the amplifying section [Fig. 1(b)] thus exhibiting gain, while the other one is loss dominated

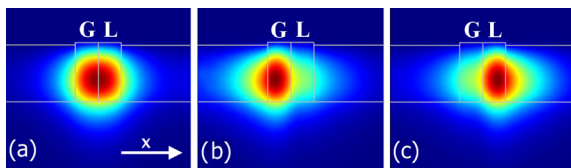


FIG. 1 (color online). A \mathcal{PT} -symmetric optical waveguide. The corresponding symmetric fundamental mode of the structure exists only below a certain critical value of the imaginary index of the waveguide (a). Above this threshold, \mathcal{PT} symmetry is lost and the fundamental mode breaks up into 2 modes: one experiencing gain (b) while the other loss (c).

[Fig. 1(c)]. In this regime, the propagation constants of these two new modes are complex conjugates of each other. The physics below and above this exceptional point is unique to non-Hermiticity and occurs in many physical situations such as, for example, in open quantum systems involving long-lived resonances [19].

\mathcal{PT} optical dynamics can also be observed in coupled wave geometries, where a \mathcal{PT} potential is formed through the interplay of two distinct gain and loss regions [13,20]. This structure is \mathcal{PT} -symmetric around its central axis. To analyze the behavior of this structure, we employ a coupled-mode formalism where the exact profiles of the isolated modes were obtained by employing a full vectorial treatment and subsequently used to evaluate their effective refractive index and the associated coupling constants via overlap integrals [21]. To do so, we express the \mathcal{PT} supermodes in terms of the individual isolated modes of the waveguides which are complex conjugate [13]. If we express the optical fields in these two channels in terms of z -dependent amplitudes $U_{1,2}(z)$, i.e., $E_n = U_n(z)F_n(x, y)$, with $F_n(x, y)$ representing the guided wave distributions in $n = 1, 2$, one finds that

$$i \frac{d}{dz} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} \beta + \delta & \kappa \\ \kappa^* & \beta^* + \delta^* \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}. \quad (2)$$

In the above equations, β and β^* represent the two propagation constants, κ is a complex coupling coefficient, and δ is a correction to the wave numbers resulting from the proximity of adjacent channels. In general, the evolution of

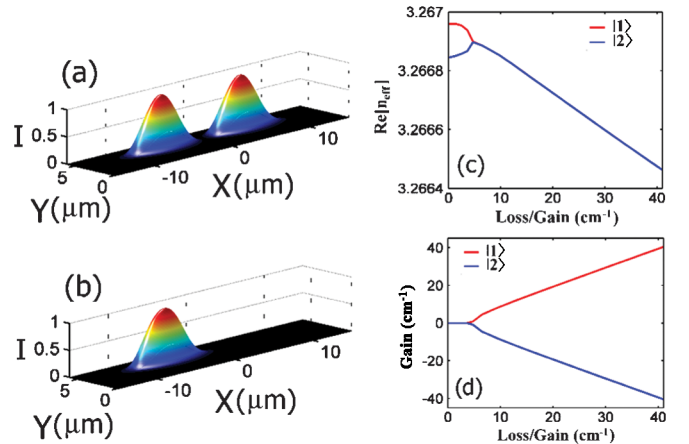


FIG. 2 (color online). Calculated supermodes of a \mathcal{PT} -symmetric optical system. (a) Below the critical gain/loss value, the modal intensity is equally divided between the two sites. As the non-Hermitian \mathcal{PT} potential strength is increased beyond the \mathcal{PT} -symmetry breaking point, the two modes become isolated in each site as shown in (b). Below the phase transition point of $\sim 3.7 \text{ cm}^{-1}$, the spectrum of the dual non-Hermitian structure is completely real, followed by a pronounced transition to the complex domain where the eigenmodes become complex conjugates (c),(d). The horizontal axis in (c) and (d) represents the gain/loss contrast in the \mathcal{PT} arrangement.

the two nonorthogonal supermodes $|1\rangle \exp(i\lambda_1 z)$ and $|2\rangle \exp(i\lambda_2 z)$ associated with Eq. (2) is given in terms of sinusoidal or exponential functions depending on whether the two eigenvalues $\lambda_{1,2} = -\text{Re}[\beta + \delta] \pm \sqrt{|\kappa|^2 - (\alpha/2)^2}$ are real or complex. In the last expression, the term $\alpha/2 = -\text{Im}[\beta + \delta]$ leads to either gain or loss in the two channels. \mathcal{PT} -symmetry breaking occurs whenever $|\alpha| \geq \alpha_c = 2|\kappa|$. This marks the onset of a phase transition beyond which the oscillatory coupling between the two modes disappears and is replaced by a hyperbolic behavior (one mode decays and the other grows). The numerically obtained intensity profile of the first supermode $|1\rangle$ below and above the transition point in a typical AlGaAs gain/loss system is shown in Figs. 2(a) and 2(b), respectively. Furthermore, the complex bifurcation of the supermode propagation constants around this point is shown in Figs. 2(c) and 2(d) for the same structure. In Figs. 1 and 2, for demonstration purposes, we have considered only purely antisymmetric gain/loss structures.

It is important to note that \mathcal{PT} -symmetry breaking can also occur in entirely passive dual systems where one channel exhibits loss ($-\text{Im}[\beta_2] \cong \alpha/2$) while the other is lossless, i.e., $\text{Im}[\beta_1] = 0$. In this new configuration,

\mathcal{PT} symmetry can be reestablished through the gauge transformation: $U_{1,2}(z) = V_{1,2}(z) \exp(-\alpha z/4)$. In this case, one can readily show that the reduced $V_{1,2}(z)$ amplitudes behave in a \mathcal{PT} fashion where the new phase transition point is now shifted to $\alpha_c \approx 4\kappa$. In fact, the transition from \mathcal{PT} to broken symmetry has a significant effect on the overall transmission behavior of this system. Normally, one intuitively expects that the transmission would drop with increasing losses. As we will see, however, after the phase transition point, this arrangement becomes anomalously transparent; i.e., its transmission increases as the loss becomes even higher. This is indeed characteristic of exceptional point behavior [19]. To analyze this behavior one can consider the two nonorthogonal \vec{V} eigenvectors of the reduced system [22]. The column eigenvectors below the phase transition point are given by $|1, 2\rangle = (1, \pm e^{\mp i\theta})$ with corresponding eigenvalues being $\mp \cos\theta$ where $\sin\theta = \alpha/4\kappa$. On the other hand, above the critical point, i.e., for $\alpha > 4\kappa$, $|1, 2\rangle = (1, -ie^{\mp\theta})$, where in this range $\cosh\theta = \alpha/4\kappa$ and the two eigenvalues are $\mp i \sinh\theta$. In the case where only the lossless channel is excited, the total transmission coefficient of this system in these two regimes is expressed by

$$T = \begin{cases} \frac{\exp(-2Z \sin\theta)}{\cos^2\theta} [\sin^2(Z \cos\theta) + \cos^2(Z \cos\theta - \theta)] & \text{for } \alpha \leq 4\kappa, \\ \frac{\exp(-2Z \cosh\theta)}{\sinh^2\theta} [\sinh^2(Z \sinh\theta) + \sinh^2(Z \sinh\theta + \theta)] & \text{for } \alpha \geq 4\kappa, \end{cases} \quad (3)$$

where in Eq. (3) $Z = \kappa z$.

To observe the effects arising from passive \mathcal{PT} -symmetry breaking, we designed a non-Hermitian passive optical double-well structure. The two waveguides were fabricated through a multilayer $\text{Al}_x\text{Ga}_{1-x}\text{As}$ heterostructure of varying concentrations [Fig. 3(a)]. The introduction of loss in the structure was carefully done in order to maintain the even real refractive index distribution necessary for \mathcal{PT} symmetry. This was achieved through deposition of a thin 100 nm layer of chromium on one of the coupler arms [Fig. 3(b)]. Chromium was intentionally chosen to overcome restrictions from the Kramers-Kronig relations since at the wavelength of operation ($\lambda_0 = 1.55 \mu\text{m}$) this metal leads to heavy losses while the detuning between the two waveguide sites is at a minimum [23]. A number of such structures were fabricated with varying Cr stripe widths, which allowed us to control the loss parameter for $\alpha > 15 \text{ cm}^{-1}$ [Fig. 3(c)], whereas nonperiodic deposition of Cr along the propagation direction allowed for effective losses below 15 cm^{-1} . The sample was set to be 16 mm, which is approximately twice the coupling length (when no loss is present). The losses were engineered to vary in the range $0\text{--}40 \text{ cm}^{-1}$ as is shown in Fig. 3(c) along with the theoretically calculated losses using finite element mode solvers.

In our experiments, vertically polarized monochromatic light at 1550 nm was coupled into the nonlossy waveguide of this structure. The total power from both channels of this system was then collected and analyzed as a function of waveguide loss. The normalized transmission results are presented in Fig. 4, indicating a strong non-Hermitian behavior. As the loss in the dual complex potential is initially increased, a decay of the total output transmission occurs as one would readily expect. However, past a critical point ($\alpha_c \approx 4\kappa \approx 9 \text{ cm}^{-1}$), namely, the passive

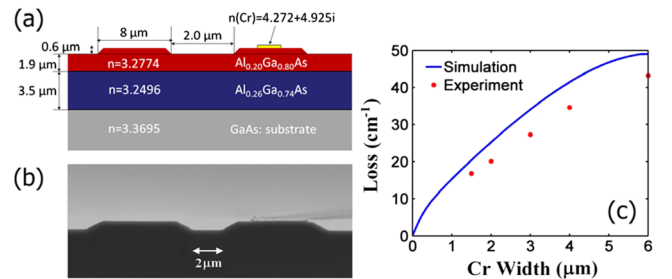


FIG. 3 (color online). Non-Hermitian dual structure. (a) Design details and complex refractive index distribution. (b) Scanning electron microscopy picture of the finalized passive \mathcal{PT} device with the Cr stripe shown on the right. (c) Modal loss of isolated waveguide structure as a function of Cr width.

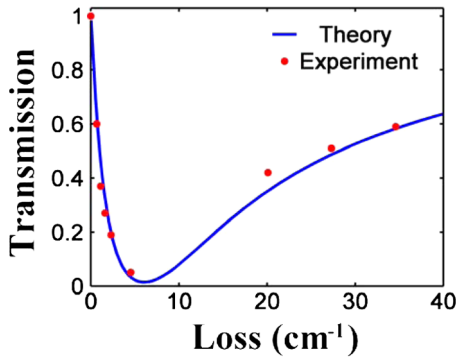


FIG. 4 (color online). Experimental observation of spontaneous passive \mathcal{PT} -symmetry breaking. Output transmission of a passive \mathcal{PT} complex system as the loss in the lossy waveguide arm is increased. The transmission attains a minimum at 6 cm^{-1} .

\mathcal{PT} -symmetry breaking point, the transmission is found to increase as the loss is increased, as also indicated by Eq. (3). Loss enhanced transmission is a direct manifestation of \mathcal{PT} non-Hermiticity. Below the transition point, light is continuously exchanged between both waveguide arms, and, as such, increasing the loss will lead to a decreased output. **At the transition point the symmetry of the structure is lost via passive \mathcal{PT} -symmetry breaking, and coupling between the waveguide channels begins to be reduced. As the losses are increased further the supermodes become increasingly different; as such, the power exchange is reduced and the total transmission is increased.** It is important to note that in our experiments the minimum transmission point differs somewhat from the calculated theoretical passive \mathcal{PT} -symmetry breaking point (9 cm^{-1}). This result is consistent with the predictions of Eq. (3), which also imply that the minimum of the intensity transmission curve does not occur precisely at α_c . We note that this \mathcal{PT} route to transparency is closely relevant to some recent suggestions concerning amplifying metamaterials. This, for example, can occur in parametric amplifiers using backward propagation in negative index structures [24,25].

In conclusion, we have demonstrated the onset of passive \mathcal{PT} -symmetry breaking within the context of optics. This phase transition was found to lead to a loss induced optical transparency in specially designed pseudo-Hermitian potentials. This work may pave the way towards the observation of other \mathcal{PT} -related features such as power oscillations and nonreciprocal wave propagation in pseudo-Hermitian lattices. The extension of these ideas in the spatiotemporal domain (e.g., optical cavities and resonators) as well as in the study of open quantum systems can be another fruitful direction.

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