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METHOD FOR SYNTHESIS OF WAVEGUIDE MODE CONVERTERS

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We propose an iterative method for synthesis of waveguide mode converters. An application of this method for bent circular waveguides is considered. Examples of synthesized converters calculated using this method are presented. The method offers fundamentally new solutions to the problems of mode converter development.

1. INTRODUCTION

The construction of waveguide transmission lines requires the use of components ensuring that waveguide modes be preserved or converted from one into another when the transmission line is bent or its parameters (e.g., radius) are varied. The synthesis methods developed to date (see, e.g., [1]) were in many respects unsatisfactory, both in regard to converter parameters, such as the length and the conversion coefficient, and computer resources. The objective of this paper is to formulate a new efficient iterative algorithm for synthesis of waveguide mode converters.

2. DESCRIPTION OF THE METHOD

Consider a system of linear differential equations of the form

$$\frac{\mathrm{d}a_j}{\mathrm{d}z} = ih_j(z)a_j + i\sum_{k\neq j}\kappa_{jk}(z)a_k. \quad (1)$$

Such a system corresponds, e.g., to the problem of one-dimensional propagation of waves in an inhomogeneous waveguide (with time-harmonic dependence $\exp(-i\omega t)$). In this case, a_j and h_j are the amplitude and wave number of the jth mode, respectively, and $\kappa_{jk}(z)$ are the coupling coefficients of the kth and jth modes. In the simplest case where all waves propagate without losses in the positive direction of the z axis, the wave numbers are positive. The eigenvalues and coupling coefficients for the modes of a hollow metal waveguide are obtained in [2, 3]. In this case, wave numbers can be considered either constant (if a waveguide with small deformations is analyzed by the perturbation method) or dependent on the z coordinate (if inhomogeneous waveguides are considered by the cross-section method). By renormalization of the amplitudes, it can be ensured that the relationship $\kappa_{jk}(z) = \kappa^*_{kj}(z)$ is satisfied. Here, the asterisk stands for complex conjugation. The energy conservation law takes the form $\sum_j |a_j|^2 = 1$.

Consider the case of propagation of all waves in the positive direction of the z axis. We assume that the total length of the converter is constant and specify two possible boundary conditions on its ends, namely, the wave amplitude vector $a_j(0)$, corresponding to the given field, at the beginning of the waveguide and the desired amplitude vector $a_j(L)$ at the end of the waveguide. It should be noted that while the first

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boundary condition is always sufficient for calculation of wave amplitudes inside the waveguide, the second one is not sufficient since the phase of each wave at the output is often determined with accuracy of the same constant. Denote the wave amplitude distribution obtained with the use of the first boundary condition as $a_j^{(1)}(z)$. If the second boundary condition is not sufficient, then the missing phase can be introduced based on the values of $a_j^{(1)}$ at the output. After this, one can obtain the wave amplitude distribution with the use of the second boundary condition. Denote this distribution as $a_j^{(2)}(z)$. In the case where the waveguide profile ensures complete conversion of the initial to the desired amplitude vector, the inequality $a_j^{(1)}(z) = a_j^{(2)}(z)$ will be satisfied for all j and z. If conversion is not complete, then, correspondingly, $a_j^{(1)}(z) \neq a_j^{(2)}(z)$.

To construct an iterative procedure for synthesis of coupling coefficients, we find the correction for coupling coefficients in each step. Such a correction is determined by the difference of two obtained distributions. We also require that the correction yield an asymptotic (for $(h_2 - h_1)\, L \gg \pi$) solution of the two-wave problem (conversion of one wave into another in the case where only two nondegenerate waves are considered) in one iteration. In the case of a real coupling coefficient, the solution is

$$\kappa_{12}(z) = \frac{\pi}{L}\, \sin[(h_2 - h_1)\, z], \tag{2}$$

where L is the total length of the converter. It can easily be verified that two conditions specified above correspond to the following expression:

$$\Delta\kappa_{12} = \frac{\pi}{2L}\, \mathrm{Im}\left[a_1^{(1)*} a_2^{(2)} - a_1^{(2)*} a_2^{(1)} + a_2^{(1)*} a_1^{(2)} - a_2^{(2)*} a_1^{(1)}\right]. \tag{3}$$

To determine the necessary deformation of the waveguide surface, we consider the simplest case where coupling coefficients are products of constants dependent on types and indices of the waves and the unique function describing the deformation (e.g., the curvature in the case of bending a regular waveguide or the tangent of the inclination angle of the waveguide generatrix in the case of axisymmetric deformation), namely,

$$\kappa_{ij}(z) = \gamma_{jk} f(z). \tag{4}$$

The correction for f takes the form

$$\Delta f_{jk}(z) = \frac{\pi}{2L}\, \mathrm{Im}\left[\frac{a_j^{(1)*} a_k^{(2)} - a_j^{(2)*} a_k^{(1)}}{\gamma_{jk}} + \frac{a_k^{(1)*} a_j^{(2)} - a_k^{(2)*} a_j^{(1)}}{\gamma_{kj}}\right]. \tag{5}$$

In calculations of waveguides with more than two waves, corrections from each pair of interacting waves are added, i.e., $\Delta f(z) = \sum_{j,k} \Delta f_{jk}(z)$. In each iteration of the algorithm, the difference of the distributions $a_j^{(1)}(z)$ and $a_j^{(2)}(z)$ for the current value of $f(z)$ is found and then the correction $\Delta f(z)$ and the corresponding change in the waveguide profile are calculated. Certain constraints can also be useful in numerical implementations of the method. Firstly, very weakly interacting waves ($\int_0^L |\kappa_{ij}(z)|\, \mathrm{d}z \ll \pi/2$) can be ignored when calculating the correction since interaction with such waves cannot give a considerable contribution to the conversion. Secondly, deformation must be limited to a certain maximum value to exclude going beyond the applicability range of the initial system and to ensure that the coupling coefficients are smaller than the waveguide eigenvalues.

3. IMPLEMENTATION OF THE METHOD FOR A BENT CIRCULAR WAVEGUIDE

In the case of a bent circular waveguide, the coupling coefficients are nonzero only for wave pairs whose azimuthal indices differ by unity. The expressions for the coupling coefficients are given in [3]. The coupling coefficients are inversely proportional to the bend radius, i.e., are proportional to the waveguide curvature, so that it is expedient to choose the latter as the function $f(z)$. Since the value of the total bend

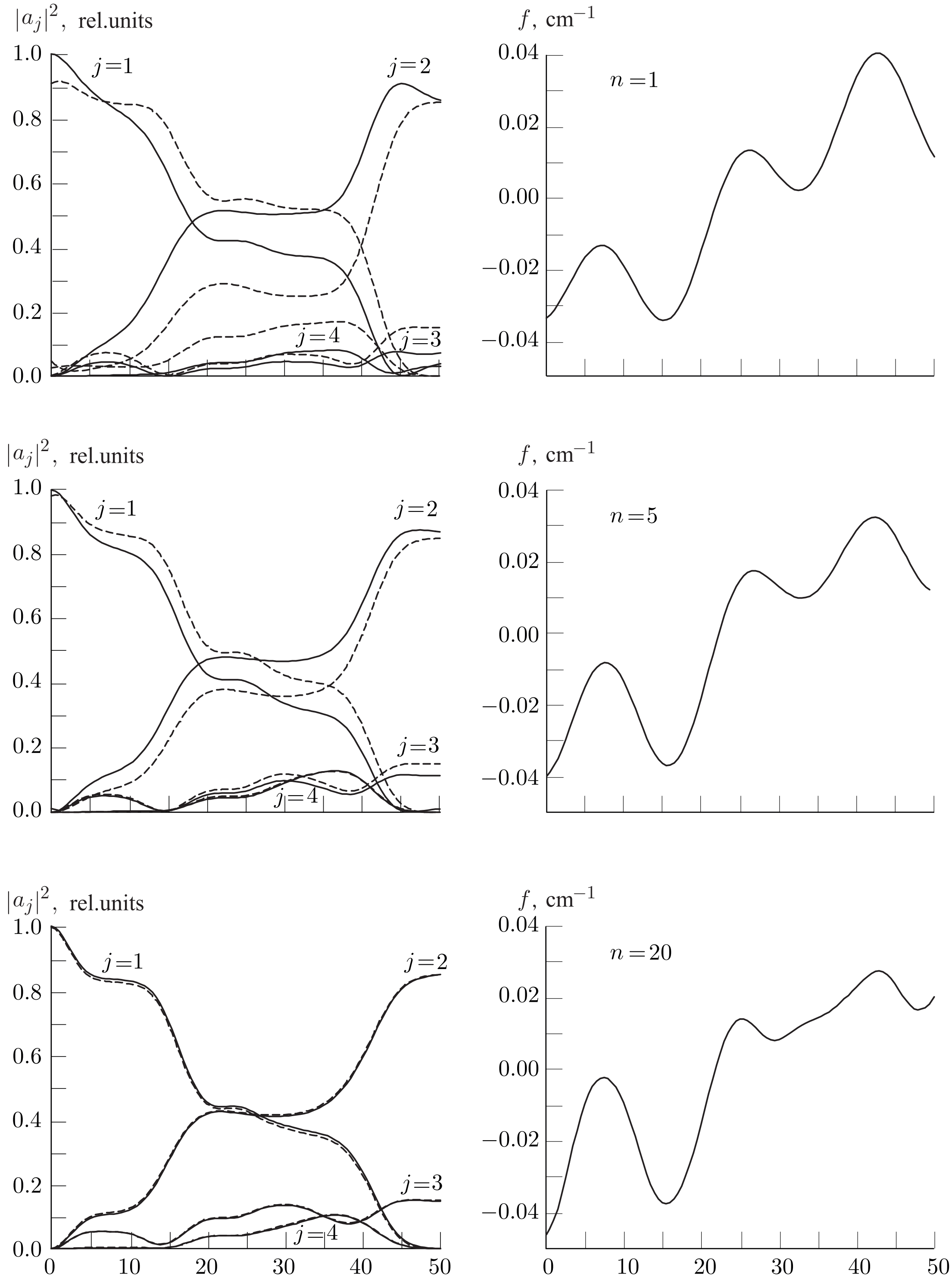

Fig. 1. Results of calculation of the converter of the TM_{01} mode to a Gaussian beam; n is the iteration number, $|a_j|^2$ is the power of the jth wave (the total power of all waves is equal to 1), and f is the waveguide curvature. The waves obtained with the use of the boundary condition on the right-hand end are shown by a dashed curve. The first wave is TM_{01}, the second is TE_{11}, the third is TM_{11}, and the fourth is TE_{21}.

angle is one of the most often specified requirements for a bent waveguide, it is necessary to provide for the possibility of assigning this parameter. The bend angle is an integral of the curvature with respect to the longitudinal coordinate, so that the total bend angle can be kept constant by adding a constant of the form

$$\Delta f_0 = \frac{1}{L}\left[\varphi_\Sigma - \int\limits_0^L f(z)\,\mathrm{d}z\right] \tag{6}$$

to the curvature after each iteration. Here, φ_Σ is the given bend angle and L is the total length of the converter. Also, imposing an upper bound on the curvature modulus is necessary even from geometric considerations since the curvature radius cannot be smaller than the waveguide radius.

4. EXAMPLES OF WAVEGUIDE CONVERTERS CALCULATED USING THE PROPOSED METHOD

On the basis of this method, we calculated various waveguide mode converters. In many cases, the results were checked by calculations of synthesized waveguides using other methods. In this paper, we give only two examples of such a synthesis.

4.1. Converter of the TM_{01} mode to a Gaussian wave beam

In this case, the purpose of synthesis of a waveguide is to obtain a mixture of TE_{11} (85% of the power) and TM_{11} (15% of the power) modes at the converter output, with phase difference equal to π. The spatial structure of this mixture is very close to the structure a linearly polarized Gaussian wave beam. Figure 1 shows the case where the wavelength $\lambda = 30$ mm, the waveguide length is 50 cm, and the waveguide diameter is $1.8\,\lambda$. In the calculations, we took into account 9 modes, i.e., all propagating modes of the given frequency which are excited by the TM_{01} mode in a bent waveguide. The maximum content of the desired wave mixture amounted to 99.98%. The total time of calculation (several tens of iterations) using a 2-GHz processor is less than one minute.

4.2. Waveguide bent by an angle of 90° with the TE_{01} mode preserved

The wavelength $\lambda = 30$ mm, the waveguide length is 50 cm, and the waveguide diameter is 1.8λ. The TE_{01} and TM_{11} modes are degenerate. For these parameters of a waveguide, the rotation angle is close to the Jouguet angle and, therefore, an almost complete undesirable conversion of the TE_{01} mode into the TM_{11} mode is reached if the waveguide curvature is constant. This is seen in Fig. 2 for the zeroth iteration ($n = 0$). However, owing to coupling with waves with are not degenerate with the TE_{01} wave (mainly, TE_{11} and TE_{12} in this case), we obtained up to 99.98% of the power in the TE_{01} mode as a result of the iterative synthesis. In the calculations, we took into account all propagating modes at the given frequency (9 modes). The total time of calculation using a 2-GHz processor is less than one minute.

Despite the strong dependence of the curvature on the longitudinal coordinate, the waveguide profile differs from the uniform bend only insignificantly (see Fig. 3) since upon double integration, which is required for obtaining the coordinates of points on the axis, each oscillating term is divided by its frequency squared. As a result, such terms considerably decrease with respect to the constant component of the curvature.

Analyzing the waveguide profile found as a result of synthesis, it can be noted that the waveguide curvature has oscillating components which provide for coupling of the TE_{01} and TM_{11} waves with certain auxiliary waves. As a result of this coupling, the wave degeneracy is removed and the necessary wave is realized at the output. This example illustrates that the proposed method permits one to find the solution of the problem even if the initial and the desired waveguide profiles strongly differ.

5. CONCLUSIONS

We have formulated a new iterative method for synthesis of the waveguide mode converters described by systems of linear differential equations. This method makes it possible to obtain converters with unique parameters.

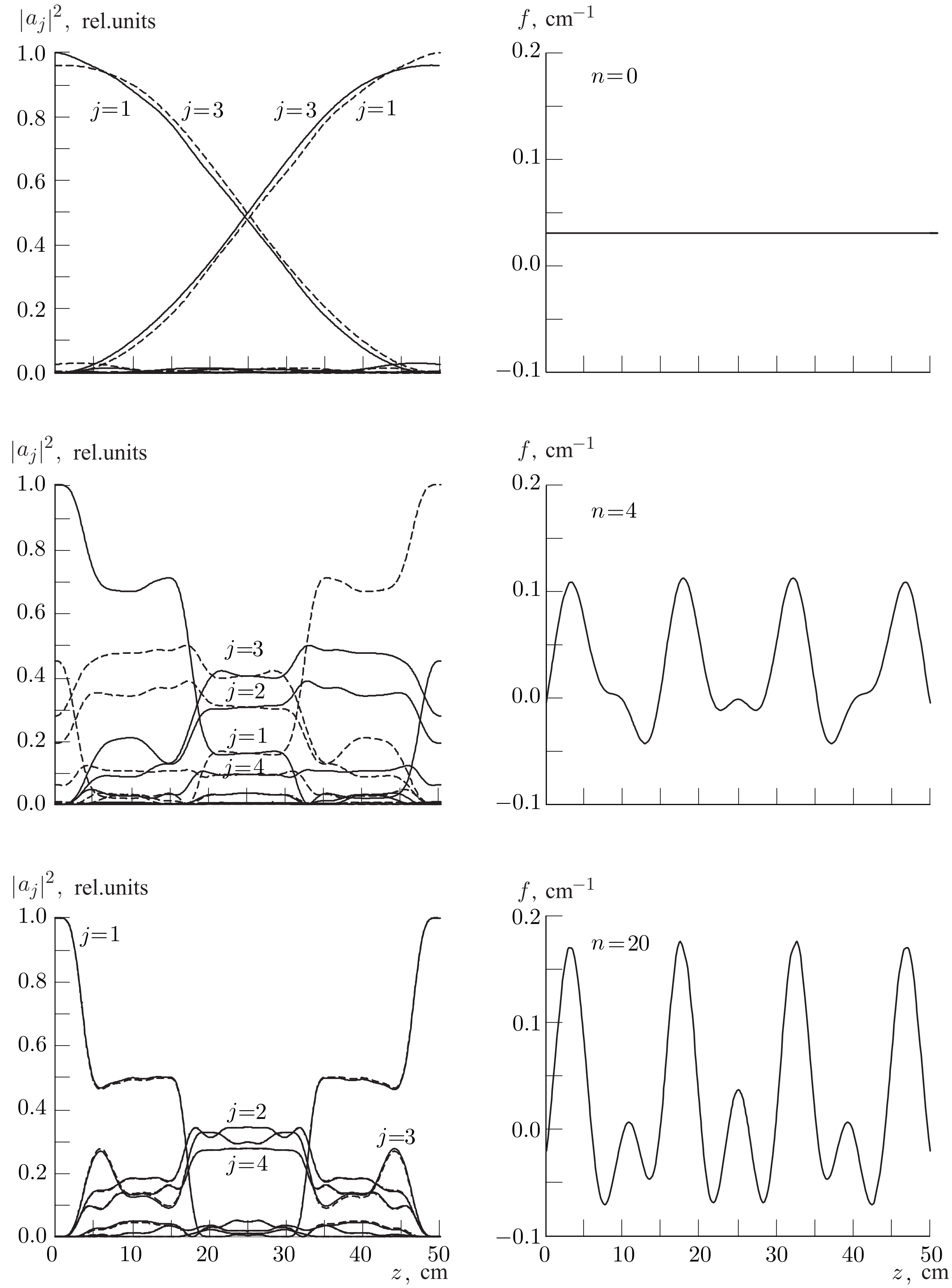

Fig. 2. Results of calculation of the waveguide bend with the TE_{01} mode preserved; n is the iteration number (the zeroth iteration is a bend with constant curvature), $|a_j|^2$ is the power of the jth wave (the total power of all waves is equal to 1), and f is the waveguide curvature. The waves obtained with the use of the boundary condition on the right-hand end are shown by a dashed curve. The first wave is TE_{01}, the second is TE_{11}, the third is TM_{11}, and the fourth is TE_{12}.

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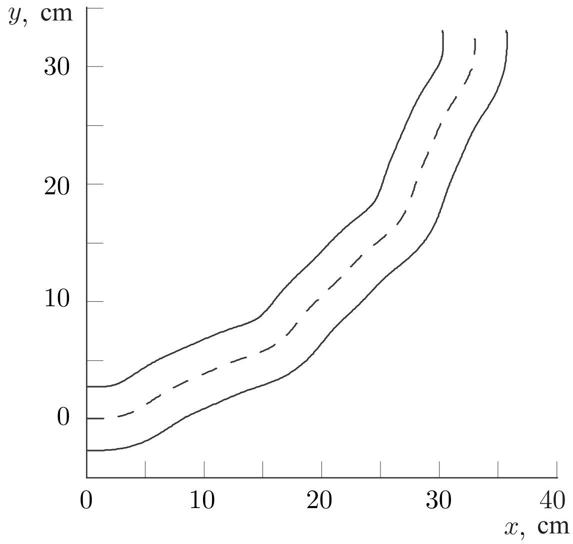

Fig. 3. Axial cross section of the waveguide bend with the TE_{01} mode preserved (this corresponds to the 20th iteration). The waveguide axis is shown by a dashed curve.

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