Magnetic Plasmon Propagation Along a Chain of Connected Subwavelength Resonators at Infrared Frequencies

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A one-dimensional magnetic plasmon propagating in a linear chain of single split ring resonators is proposed. The subwavelength size resonators interact mainly through exchange of conduction current, resulting in stronger coupling as compared to the corresponding magneto-inductive interaction. Finite-difference time-domain simulations in conjunction with a developed analytical theory show that efficient energy transfer with signal attenuation of less then 0.57 dB/μm and group velocity higher than $1/4c$ can be achieved. The proposed novel mechanism of energy transport in the nanoscale has potential applications in subwavelength transmission lines for a wide range of integrated optical devices.

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A fundamental problem of integrated optics is how to transport electromagnetic (EM) energy in structures with transverse dimensions that are considerably smaller than the corresponding wavelength of illumination. The main reason to study light guiding in the nanoscale has to do with the size of transmission lines being a limiting factor for substantial miniaturization of integrated optical devices. Planar waveguides and photonic crystals are currently key technologies enabling a revolution in integrated optical components [1,2]. However, the overall size and density of the optical devises based on these technologies is limited by the diffraction of light, which sets the spatial extend of the lowest guided electromagnetic mode at about half wavelength.

Recently, a new method of electromagnetic energy transport has been proposed that allows size reduction of the optical devices to below the diffraction limit [3–5]. The EM energy is coherently guided via an array of closely spaced metal nanoparticles due to a near-field coupling. Metal particles are well known to support collective electronic excitation, surface plasmon (SP) with resonance frequencies depending on the particle size and shape. Owing to the SP resonances, metal nanoparticles exhibit strong light absorption with absorption cross section far exceeding their geometrical sizes. Thus, metal nanostructures could efficiently convert EM energy into oscillatory electron motion, which is a necessary condition for strong coupling of light into a waveguiding structures.

In 1999, [6] Pendry reported that nonmagnetic metallic element, double split ring resonator (DSRR), with size below the diffraction limit, exhibits strong magnetic response and behaves like an effective negative permeability material. While in such systems, there are no free magnetic poles; the excitation of displacement currents in the DSRR results in induction of a magnetic dipole moment that is somehow similar to a bar magnet. In analogy to the SP resonances in metal nanoparticles, an effective media made of DSRRs could support resonant magnetic plasmon (MP) oscillations at GHz [6–8] and THz frequencies [9–12]. Combined with an electric response, characterized by negative permittivity, such systems could lead to development of metamaterials with negative indexes of refraction [7,8].

In this Letter, we propose a subwavelength size metal structure, referred to as a single split ring resonator (SSRR) which (a) demonstrates magnetic resonance in the THz range, and (b) could be used to support propagation of long range MP polaritons. It is well known that radiation loss of a magnetic dipole is substantially lower as compared to the radiation of an electric dipole of similar size [13]. Thus, application of MP for guiding EM energy for long distances has great potential for direct application in novel subdiffraction size transmission lines. Indeed, magnetic plasmons have been already shown to play an important role in the excitation of magneto-inductive [14] and electro-inductive waves [15] in the microwave range. In this Letter, we show that at high frequencies, a coupling mechanism based on exchange of conduction current between specially designed resonators may be utilized to efficiently transfer energy along a subwavelength sized metal nanostructures. The interaction due to the conduction current is found to be much stronger than the corresponding magneto-inductive coupling, which leads to significant improvement in the properties of the guided MP wave.

Figure 1(a) presents a novel design of a SSRR characterized by two half-space metal loops with tails adjacent to their ends. Pendry's double split ring structure has no tail; nevertheless, the space between the rings acts as a capacitor allowing the flow of a displacement current [6]. In the present design, it is the gap in the tails that plays the role of a capacitor. Excitation of magnetic response in a system of SSRRs fabricated on a planar substrate results in induction of magnetic dipoles moments that are perpendicular to the

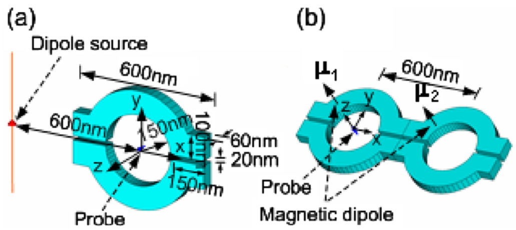

FIG. 1 (color online). Schematic illustration of a single split ring resonator (a) and a connected unit for implementation in subwavelength sized MP transmission lines (b). The geometrical characteristics and position of the dipole source excitation are included.

substrate plane (see Fig. 1). Parallel dipoles are characterized with small spatial field overlap, and consequently the magneto-inductive interactions between them are expected to be rather weak. To substantially increase coupling between the dipoles, we physically connect the SSRRs, as shown in Fig. 1(b). The contact between the rings serves as a ''bond'' for conduction current to flow from one SSRR to its neighbor. Thus, in addition to the magneto-inductive coupling, our system interacts directly by exchange of conduction current. This type of coupling is somewhat similar to the electron exchange interaction between two magnetic atoms in a ferromagnetic material [16]. As shown below, the direct physical link between the resonators leads to stronger interaction between the SSRRs and improves the EM energy transport along a chain of SSRRs.

To study the EM response of the proposed SSRR, we perform a set of finite-difference time-domain (FDTD) calculations using a commercial software package CST Microwave Studio (Computer Simulation Technology GmbH, Darmstadt, Germany). In the calculations, we rely on the Drude model to characterize the bulk metal properties. Namely, the metal permittivity in the infrared spectral range is given by $\varepsilon(\omega) = 1 - \omega_p^2/(\omega^2 + i\omega\omega_\tau)$, where ω_p is the bulk plasma frequency and ω_τ is the relaxation rate. For gold, the characteristic frequencies fitted to experimental data are $\hbar\omega_p = 9.02$ eV and $\hbar\omega_\tau = 0.027$ eV [17]. In the numerical calculations, a dipole source excites the SSRR, and a probe is employed to detect the local field at the center of SSRR (see Fig. 1). In the case of a single SSRR, we obseve a well pronounced resonance at $\hbar\omega_0 = 0.36$ eV with resonance bandwidth $\hbar\Gamma = 10$ meV (see Fig. 2). Introduction of a second SSRR, shown in Fig. 1(b), results in splitting of the MP resonance due to interaction. The two nondegenerate MP modes have eigenfrequencies $\hbar\omega_1 = 0.24$ eV and $\hbar\omega_2 = 0.38$ eV, respectively.

To better understand the interactions involved in the splitting of the MP resonance, we develop a comprehensive semianalytic theory based on the attenuated Lagrangian formalism. If q_m is the total oscillation charge in the m-th SSRR, L is the induction of the ring, and C is the capacitance of the gap, then we can write the Largangian of the

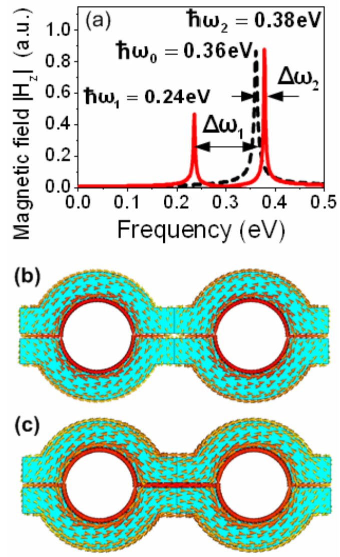

FIG. 2 (color online). (a) The magnetic field amplitude $|H_z|$ detected at the center of the resonators is plotted vs excitation frequency. Frequency split $\Delta\omega_{1,2}$ is observed for the case of two physically connected SSRRs (solid line). The local current distributions are calculated for the (b) antisymmetric and (c) symmetric MP modes.

coupled system as

$$\Im = \frac{1}{2}L(\dot{q}_1^2 + \dot{q}_2^2) - \frac{1}{2C}(q_1^2 + q_2^2) + M\dot{q}_1\dot{q}_2 - \frac{1}{4C}(q_1 - q_2)^2 \tag{1}$$

where the first two terms correspond to the energy stored in the inductors and end capacitors, the interaction term $M\dot{q}_1\dot{q}_2$ is due to magneto-inductive coupling, and the static energy stored in the shared middle capacitor is also included. By introducing Ohmic dissipation $\Re = \frac{1}{2}\gamma(\dot{q}_1^2 + \dot{q}_2^2)$ and substituting Eq. (1) in the Euler-Lagrangian equations

$$\frac{d}{dt}\left(\frac{\partial\Im}{\partial\dot{q}_m}\right) - \frac{\partial\Im}{\partial q_m} = -\frac{\partial\Re}{\partial\dot{q}_m}, \qquad m = 1, 2, \tag{2}$$

we obtain a coupled equations for the magnetic moments $\mu_m = A\dot{q}_m$ (A is a constant related to the area of SSRR and it geometry)

$$\ddot{\mu}_1 + \omega_0^2\mu_1 + \Gamma\dot{\mu}_1 = \frac{1}{2}\kappa_1\omega_0^2(\mu_1 + \mu_2) - \kappa_2\ddot{\mu}_2 \tag{3a}$$

$$\ddot{\mu}_2 + \omega_0^2\mu_2 + \Gamma\dot{\mu}_2 = \frac{1}{2}\kappa_1\omega_0^2(\mu_1 + \mu_2) - \kappa_2\ddot{\mu}_1, \tag{3b}$$

where $\omega_0^2 = 2/(LC)$ and $\Gamma = \gamma/L$ are the degenerated MP mode eigenfrequency and bandwidth, respectively. Clearly, the electromagnetic coupling between the resonators is governed by two separate mechanisms. The first term to the right side of Eqs. (3), corresponds to interaction due to exchange of conduction current while the second term is the magneto-inductive contribution. The coupling coefficients are related to the equivalent circuit characteristics of the SSRR. For instance, $\kappa_2 = M/L$ depends on the SSRR's mutual and self inductance and for an ideal circuit $\kappa_1 = 1/2$ [see Eq. (1)].

Eqs. (3) yield solutions in the form of damped harmonic oscillations $\mu = \mu_{i0}\exp(-\frac{1}{2}\Gamma_i t + i\omega_i t)$, where the index $i = 1, 2$ specifies the MP mode. Using that $\Gamma/2\omega_0 \ll 1$, it is straightforward to estimate from Eqs. (3) the system eigenfrequencies as $\omega_1 = \omega_0\sqrt{(1-\kappa_1)/(1+\kappa_2)}$ and $\omega_2 = \omega_0/\sqrt{1-\kappa_2}$, where the coupling coefficients $\kappa_1 \approx 0.5$ and $\kappa_2 \approx 0.1$ are retrieved numerically. For the high frequency (antisymmetric) mode ω_2, one has $\mu_1 = -\mu_2$, and the exchange current interaction term in Eq. (1) is negligible. Consequently, the observed frequency shift $\Delta\omega_2$ is predominantly due to the magneto-inductive coupling between the SSRRs. This phenomenon is depicted in Fig. 2(b), where we have plotted the local current density inside the resonators. Two distinctive current loops, each closed through a displacement current at the resonator tails, are formed. No conduction current is shared between the SSRRs. The opposite is true for the low-frequency (symmetric) MP mode ω_1, where $\mu_1 = \mu_2$, and both exchange of conduction current and magneto-inductive interactions contribute to the frequency shift $\Delta\omega_1$. The unimpeded flow of current between the SSRRs is clearly seen in Fig. 2(c). Comparison between the frequency shifts $\Delta\omega_1 \gg \Delta\omega_2$ and the absolute value of the coupling constants $\kappa_1 \gg \kappa_2$ undoubtedly shows that the exchange of conduction current is the dominant coupling mechanism for the proposed SSRRs system.

The magnetic dipole model described above can also be applied to investigate a finite or infinite chain of connected SSRRs. Indeed, if a magnetic dipole μ_m is assigned to each resonator and only nearest neighbor interactions are considered then the Lagrangian and the dissipation function of the system can be written as

$$\Im = \sum_m \left(\frac{1}{2}L\dot{q}_m^2 - \frac{1}{4C}(q_m - q_{m+1})^2 + M\dot{q}_m\dot{q}_{m+1}\right)$$
$$\Re = \sum_m \frac{1}{2}\gamma\dot{q}_m^2. \tag{4}$$

Substitution of Eq. (4) in the Euler-Lagrangian equations yields the equations of motion for the magnetic dipoles

$$\ddot{\mu}_m + \omega_0^2\mu_m + \Gamma\dot{\mu}_m = \frac{1}{2}\kappa_1\omega_0^2(\mu_{m-1} + 2\mu_m + \mu_{m+1}) - \kappa_2(\ddot{\mu}_{m-1} + \ddot{\mu}_{m+1}). \tag{5}$$

The general solution of Eq. (5) corresponds to an attenuated MP wave: $\mu_m = \mu_0\exp(-m\alpha d)\exp(i\omega t - imkd)$, where ω and k are the angular frequency and wave vector, respectively, α is the attenuation per unit length, and d is the SSRR's size. By substituting $\mu_m(t)$ into Eq. (5) and working in a small damping approximation ($\alpha d \ll 1$), simplified relationships for the MP dispersion and attenuation are obtained

$$\omega^2(k) = \omega_0^2\frac{1-\kappa_1[1+\cos(kd)]}{1+2\kappa_2\cos(kd)} \tag{6a}$$

$$\alpha(\omega) = \frac{\omega\Gamma}{\kappa_1\omega_0^2 + 2\kappa_2\omega^2}\frac{1}{\sin(kd)\cdot d}. \tag{6b}$$

The range of applicability and overall accuracy of the predicted relationships are compared in Fig. 3 to FDTD results for a finite chain of SSRRs. The chain size is restricted to 50 resonators lengths, which assure reliable estimates of the system properties without imposing overwhelming computational constraints. The MP polariton is excited by a dipole source placed at a distance of 600 nm from the center of the leading SSRR element, and the H field along the chain is analyzed to determine the wave vector k of the propagating mode. Numerically and analytically estimated MP dispersion and attenuation curves are depicted in Figs. 3(b) and 3(c), respectively.

In contrast to the EP polariton in a linear chain of nanosized metal particles, [3–6] where both transverse and longitudinal modes could exist, the magnetic plasmon is exclusively a transversal wave. It is manifested by a single dispersion curve (black solid line in Fig. 3(b)] which covers a broad frequency range $\omega \in (0, \omega_c)$, with a cutoff frequency $\hbar\omega_c \approx 0.4$ eV. The precise contribution of each coupling mechanism in the MP dispersion can be studied readily using Eq. (6a). Exclusion of the magneto-inductive term results in slight decrease in the cutoff frequency $\omega_c \to \omega_0$ [dashed curved line in Fig. 3(b)]. On the other hand, if the SSRRs interact only through the magneto-inductive force, a dramatic change in the MP dispersion is observed

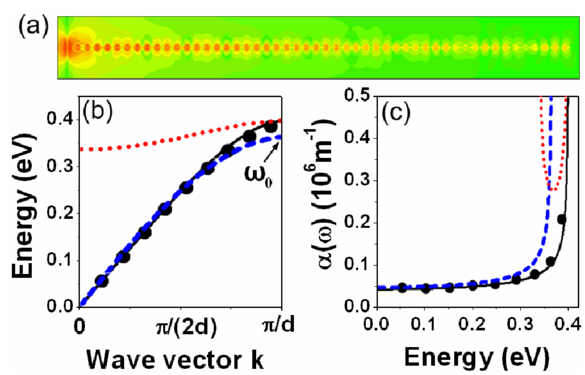

FIG. 3 (color online). (a) FDTD simulation of a MP propagation along a connected chain of 50-SSRRs at $\hbar\omega = 0.3$ eV; (b) dispersion $\omega(k)$, and (c) attenuation coefficient $\alpha(\omega)$. The analytical result [Eq. (6)] including conduction current and magneto-inductive interactions, black solid curve, matches well with the FDTD numerical data (circle dots) The predicted MP characteristics based singularly on exchange current interactions ($\kappa_2 = 0$) or magneto-inductive interactions ($\kappa_1 = 0$) are presented with dashed and dotted curved lines, respectively.

[dotted curved line in Fig. 3(b)]. Namely, the propagating band shrinks to a very narrow range of frequencies $\Delta\omega \cong 2\omega_0\kappa_2$ centered around ω_0. Such relatively short bandwidths are characteristic for the EP [5] and follows from the rapid fall of the magneto-inductive force with the distance.

Strong wave dissipation has been one of the major obstacles for utilization of surface plasmons in optical devices. The subdiffraction-sized MP transmission line, proposed in this Letter, promises a considerable improvement in the wave transmission as shown in Fig. 3(c). For most of the propagation band, $\alpha(\omega)$ stays constant and has relatively low value. For instance, at an incident frequency $\hbar\omega = 0.3$ eV, the MP attenuation coefficient is $\alpha = 0.65 \times 10^5$ m^{-1} (signal attenuation 0.57 dB/μm), which gives a field decay length of 15.4 μm (3.7 free space wavelengths) or 25.7 unit cells. For comparison, the gold nanoparticle system presented in reference [5] manifests at $\omega = 2.4$ eV, a field decay length of about 410 nm (signal attenuation 21.4 dB/μm) which also corresponds to 5.4 unit cells or 0.8 free space wavelengths. Thus, the proposed MP transmission line performs better compared to the EP not only in terms of the absolute value of the propagation length, but also in its relation to the operation free space wavelength and the size of each individual resonator. The reason behind this improvement in MP transmission is easily understood by looking at the expected attenuation when one of the coupling mechanisms is artificially impeded. Clearly, MPs excited entirely by inductive coupling, similarly to the EPs, exhibit strong attenuation, while introduction of direct physical link between the resonators improves transmission (dashed and dotted curved lines in Fig. 3(c)]. This effect is also manifested in the MP group velocity $v_g = \partial\omega/\partial k$, which reaches values up to 0.25c at the center of the propagation band, and is a factor of 4 faster than the result reported for EP [5]. Thus, compared to the EP, a MP pulse could travel at higher speeds and propagates greater distances.

Finally, it is important to mention that the MP properties can be tuned by changing the material used and the size and shape of the individual SSRRs. For instance, at $\omega = 0.3$ eV, utilization of silver instead of gold results in longer MP's field decay length of about 16.6 μm. However, silver is easily oxidized, which make MP transmission lines based on this metal less versatile and difficult to integrate with the current CMOS technology. From our simulations, not presented here, it is also clear that system size manipulation such as down scaling or change of the capacitor gap width has strong effect on the magnetic response. Generally, the MP resonance frequency increases linearly with the decrease of the overall SSRR size. Unfortunately, for high frequencies ($\hbar\omega > 1.2$ eV), this scaling tends to saturate as shown in Ref. [18]. An alternative way is to employ more complicated in shapes magnetic resonators or to change the dielectric constant of the surrounding media. All those prospective solutions and their effect on the MP propagation require further studies.

In conclusion, we have proposed and studied a one-dimensional magnetic plasmon propagating in a linear chain of novel single split ring resonators. We show that at infrared frequencies, a coupling mechanism based on exchange of conduction current could be used to improve energy transmission. A comprehensive analytical model is developed for calculation of MP dispersion, attenuation coefficient, and group velocity. The theory is consistent with the performed FDTD simulations, representing a direct evidence of effective energy transfer below the diffraction limit. Excitation of MP could be a promising candidate for the development of a wide range of optical devices, including in-plane, CMOS compatible subwavelength optical waveguides, fast optoelectronic switches, and transducers.

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